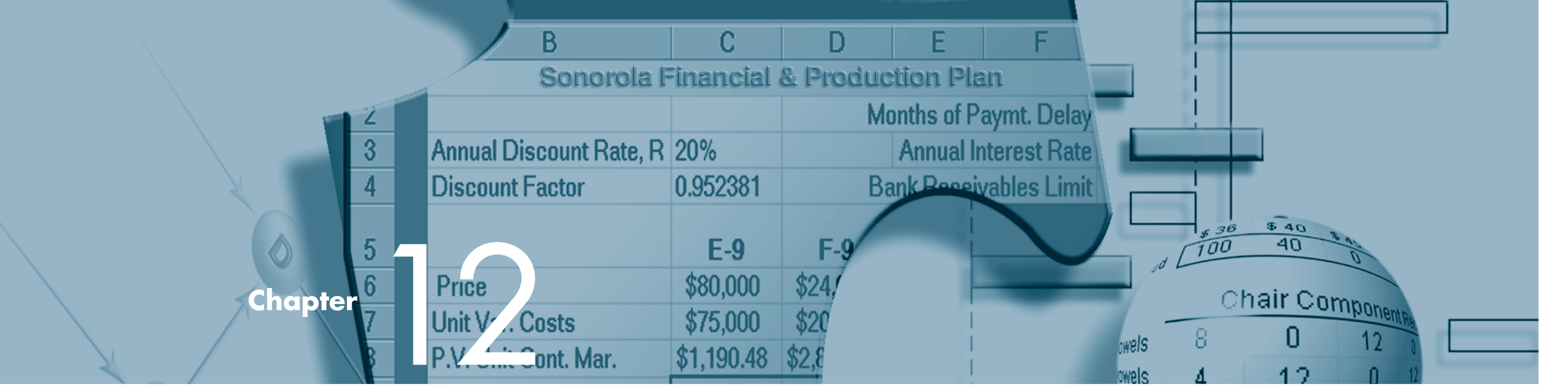


Chapter 12

MULTI-OBJECTIVE DECISION MAKING AND HEURISTICS

CHAPTER OUTLINE

APPLICATION CAPSULE **Reconciling Many Competing Objectives—Forest Land Management Issues**

Over the last two decades, several factors have altered the practice of forest land management around the world. As the population and resource development increase, many forest-based outputs are approaching or exceeding sustainable levels of use. People are increasingly aware of the need to preserve forest ecosystems, to sustain a wide spectrum of resources, and to protect threatened and endangered species, wildlife habitat, scenic beauty, and biodiversity. As a result, forest land managers are shifting their emphasis from the production of goods and services (the agricultural model) toward maintaining forest health, biodiversity, and productivity (the ecosystem model). With increased public participation in the management of all foreast resources, it is not surprising that the decision-making process has become more open, political, and complex.

Publicly owned forests have multiple uses and are subject to many concerns. Some forest outputs have direct economic value: timber, forage, recreation, and water. Others have less measurable value: scenic beauty, undisturbed wilderness landscapes, biodiversity, and preservation of endangered species. Also important are the social and economic issues of forest-dependent communities, intergenerational equity, efficiency, and fairness. The last decade has seen the growing involvement of ecologically minded groups, supported by a public increasingly concerned with environmental issues. Explicit recognition of multiple objectives in forest-planning models is becoming increasingly important.

The two multiple-objective models most widely used in forest management are based on goal programming and multiple-objective linear programming. Generally, these models optimize a given set of forest-management decisions in light of multiple objectives.

Many multiple-objective forest-management problems are resolved in an adversarial environment in which regulatory constraints are proposed in an effort to achieve satisfactory levels of hard-to-value common property resources, such as water, fish, and wildlife. Compromise solutions are being developed around the bargaining table with multiple-objective computer models helping to frame these discussions. (See Weintraub and Bare.)

12.1 INTRODUCTION

From time to time a manager's model may be so complex that the mathematical model constructed to attack it cannot be solved with the traditional algorithms available to the analyst. This situation may occur because

1. The model, "correctly formulated," may be too large, too nonlinear, or logically too complex (requiring, for example, the use of many 0–1 variables in the formulation).
2. It is felt that the imposition of simplifying assumptions or approximations, which might make the model more tractable, would destroy too much of the important real-world structure of the model (i.e., would carry the model too far from reality to be useful).

Here is a real dilemma. The model at hand is too complex to solve. At the same time we are unwilling to simplify it in any ameliorative way. What does one do in this seemingly hopeless situation?

In part to answer this question, the field of heuristic programming has developed. In the discussion above, when we employed the phrase "the model is too complex to *solve*," we were using the word *solve* in a rigorous mathematical sense. We meant that the mathematical model was so complicated that, although a rigorous solution exists (e.g., an optimal solution in an optimization model), it is too difficult, too time-consuming, perhaps even impossible to discover with existing know-how and technology. In such a case a *heuristic algorithm* might be employed.

A **heuristic algorithm** is one that efficiently provides good approximate solutions to a given model. Often (but by no means always) in employing such an algorithm one may be able to measure precisely the "goodness" of the approximation. For example, in the optimization context, with some heuristic algorithms one can make a statement like "Upon termination you can be sure of being within _% of optimality." Or, "Under certain assumptions the heuristic answer will be optimal ___% of the time." An important aspect of a heuristic algorithm is that it never gives a "bad" solution. It is more important always to have a fairly good solution than to have the best solution sometimes and a bad solution once in a while.

The term **heuristic** is also frequently encountered. A **heuristic** is an intuitively appealing rule of thumb for dealing with some aspect of a model. A collection of heuristics, or heuristic algorithms, is referred to as a **heuristic program**. Some computer codes to solve linear programs (like Excel's Solver), for example, employ heuristics in the initial phase of the simplex method to attempt to quickly find an initial corner. Heuristics are employed to get a quick start with the transportation algorithm, and so on.

As you can infer from the definitions above, you no doubt use heuristics frequently in everyday problem solving. You go to the bank, and to minimize your time waiting, you stand in the shortest line. Although this is by no means guaranteed to be optimal, it is a rule of thumb that often works quite well. In checking through customs you may prefer the bench occupied by a smiling officer, although he is certainly not guaranteed to be more lenient than others. The list goes on.

In the context of mathematical programming, heuristics are often employed in conjunction with, or as a special case of, more general or more rigorous problem-solving strategies. The important point to remember is that a heuristic procedure or algorithm is intuitively appealing but can only guarantee its results, if at all, in a statistical manner or within certain margins of uncertainty. It is employed mainly for efficiency—namely, to produce quickly what are hopefully good, if not optimal, results.

Generally speaking, from the viewpoint of a manager, a heuristic procedure may certainly be as acceptable as, and possibly (in terms of cost) even preferable to, a "more exact" algorithm that produces an optimal solution. The dominant considerations should be the amount of insight and guidance that the model can provide and the overall net benefit as measured by the difference "savings due to the model less cost of producing the model and its solution."

In the first part of this chapter we discuss several examples of heuristic algorithms as applied to large *combinatorial optimization* models. The term **combinatorial optimization** means there are only a finite number of feasible alternatives, and if all of these are enumerated, the optimal one can be found. The problem is that in practice this finite number often amounts to millions or even billions of possibilities, and hence even on high-speed computers complete enumeration is out of the question. Although such models can often be formulated as integer programs with 0–1 variables, they are often so large that even the IP formulation is prohibitively expensive to bring to optimality with the usual branch-and-bound or partial enumeration approach.

Following the examples in the first part of the chapter, we then look at models for which the objective is to achieve acceptable levels of certain "goals." For example, consider a model with multiple but conflicting objectives. The president of a firm wants high profits but also wants to maintain low prices in order to keep from losing clients. An executive with a fixed budget wants to invest in R&D to provide long-term benefits to the firm, but also wants to purchase raw materials to make a product that generates shorter-term profits. Such examples of multiple but conflicting objectives are typical in business applications. *Goal programming* deals with such models. The topic is closely related to heuristic programming, for in a sense goal programming itself could be thought of as a heuristic approach to dealing with multiple objectives.

Next, we will look at a hot new area called analytic hierarchy process (AHP), which is a tool to help managers choose between many different decision alternatives when there are multiple criteria that are used to score the alternatives. Numerous examples can be thought of that fit this general modeling area; for example, choosing a new computer or software package, selecting a university to attend, or which job offer to accept. AHP brings good discipline to this decision methodology.

12.2 FACILITY SCHEDULING (SEQUENCING JOBS)

SEQUENCE-DEPENDENT SETUP TIME

Consider a single production facility through which numerous jobs must be processed—for example, a computer, a drill press, or an ice cream machine. Typically, the facility may have to shut down after processing one job in order to set up for the next. Such "downtime" is termed **setup**, or *changeover*, **time**. The length of the setup time may depend on the next job to be processed and the job just completed. A sequence of similar jobs (making French vanilla ice cream after New York vanilla) would be interrupted by less setup time (cleaning out the machine) than a sequence of dissimilar jobs (French vanilla after Dutch chocolate). A typical managerial problem would be to *sequence the jobs in such a way as to minimize total setup time.* This is a typical problem for a company such as Monsanto Chemical, which makes chemicals in common vats or transports them by tank cars. It makes a difference in which order the chemicals are produced or transported, as to the cost of changeover.

You can easily see that from the combinatorial point of view this can be a very large model. If there are only three jobs to be processed, say jobs A, B, and C, then any of the three could be taken first, with either of the remaining two second and the third determined (i.e., the single remaining job). The possible sequences can be displayed as a tree with each branch representing one sequence. The six possibilities are shown in Figure 12.1. In general, with n jobs, there are $n! = n(n-1)(n-2)\ldots 1$ possible combinations or sequences ($n!$ is the mathematical term n factorial). Only 10 jobs produces $10! = 3{,}628{,}800$ different sequences. You can see that this number of possible sequences ($n!$) increases rapidly with the size of n.

Obviously, one way to think about solving the minimization problem above is by complete enumeration. That is, generate each of the $n!$ possible sequences of jobs and compute the total setup time required for each sequence. Then pick the sequence associated with the smallest total time. Although this algorithm would provide a true optimum, it is not practical even for modest values of n because of the large number of sequences that would have to be enumerated. If there were 20 jobs, and a computer (like IBM's Deep Blue) could calculate 6,500,000,000 combinations each second (6500 MIPS in computer language), it would take over 11 years to determine the optimal answer by listing every possible combination.

HEURISTIC SOLUTIONS

Heuristic rules, although they will not guarantee an optimal solution, are often applied to this model, for they will usually lead quite quickly to a satisfactory solution.

As an example, consider a machine operator who has three rather long batch jobs to be run on Monday afternoon. The machine is currently idle. For each of these jobs there is a setup time (cleaning the machine from the last job, setting up the individual components and other auxiliary equipment for the new job, etc.) as specified in Figure 12.2. Since there are only $3! = 3 \cdot 2 = 6$ possible sequences, they can all be enumerated. The results

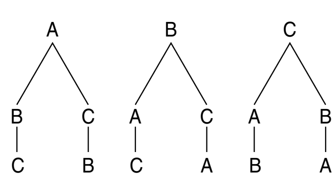


FIGURE 12.1

Tree Showing Six Possible Sequences for Three Jobs A, B, C

Table 12.1

Results of Complete Enumeration

SEQUENCE	SETUP TIME	TOTAL (MIN)
$0 \to A \to B \to C$	27 + 35 + 46	108
$0 \to A \to C \to B$	27 + 22 + 12	61
$0 \to B \to C \to A$	21 + 46 + 46	113
$0 \to B \to A \to C$	21 + 49 + 22	92
$0 \to C \to A \to B$	32 + 46 + 35	113
$0 \to C \to B \to A$	32 + 12 + 49	93

From job \ To job	A	B	C
0	27	21	32
A		35	22
B	49		46
C	46	12	

FIGURE 12.2

Setup Times in Minutes

appear in Table 12.1. As you can see, the optimal (minimum total setup time) sequence is $0 \to A \to C \to B$.

A Greedy Heuristic Let us now see how a heuristic rule might be applied to this model. The rule we shall illustrate is called the **next best rule**, sometimes called a **greedy algorithm**. The rule goes as follows:

1. At step 1 (e.g., in selecting the first job), perform the task with least initial setup time.
2. At each subsequent step, select the task with least setup time, based on the current state.

Let us now apply this rule to the data in Figure 12.2. The task with the least initial setup time is B. Hence, the first step is $0 \to B$. According to the greedy algorithm, given that we have just completed B, the task to be selected is C, since the setup for $B \to C$ is less than for $B \to A$. Thus, we have $0 \to B \to C$, and we can then finish only with A. Thus, we obtain

$$\text{greedy heuristic: } 0 \to B \to C \to A$$

$$\text{total setup time} = 21 + 46 + 46 = 113$$

Notice that this is far from optimal. In fact, in this example, the greedy heuristic, although intuitively appealing, provides the worst possible policy for our model. Although it is true that in general, for sequential decision models, the greedy algorithm does *not* lead to an optimal solution, there are in fact a few special models for which it does. (See, for example, the problem of finding a minimal spanning tree in Chapter 5.) However, the rule is extremely easy to apply, and studies on this type of model have shown that *statistically*, for the above type of sequencing model, the rule is not bad. For example, one article [see Gavett] showed that the heuristic will often produce better results than could be obtained by a purely random selection of tasks.

A Better Heuristic The same article shows that the following modified heuristic gives even better results:

1. Transform the original data in Figure 12.2 by subtracting the minimum setup time in each column from all other entries in that column. This process produces the data in Figure 12.3.
2. Apply the greedy algorithm to this set of transformed data. Doing this, we obtain

Best first step	$0 \to A$
Best second step	$A \to C$
Third step	$C \to B$

and thus the modified heuristic produces the sequence $0 \to A \to C \to B$, which was already shown to be optimal for this model.

	A	B	C
0	0	9	10
A		23	0
B	22		24
C	19	0	

FIGURE 12.3

Transformed Data

Although this modified heuristic will not always give the optimal solution, it is easy to implement, and in practice, for large models, it often produces quite good results.

12.3 SCHEDULING WITH LIMITED RESOURCES (WORKLOAD SMOOTHING)

Imagine a sequence of activities to be scheduled in order to complete a project. Basic models such as PERT and CPM, discussed in Chapter 14, will schedule the activities in such a way as to minimize total project completion time subject to the constraint that some activities cannot start until others have been completed. The resources (money, labor, machinery, and so on) needed to complete the individual activities are often considered to be available in any quantities required by any particular schedule. In reality, however, such resources may be limited, in which case resource availability becomes another constraint.

A SIMPLE EXAMPLE

As a simple example, consider the scheduling model shown in Figure 12.4 and Table 12.2. Figure 12.4 shows **precedence relationships** among the various activities. That is, it shows which activities must be completed before others can begin. For example, activity VIII cannot begin until VII is completed, and VII cannot begin until I is completed. Table 12.2 shows the duration of each activity (in weeks) and the resources required (number of people) to complete each activity.

Ignoring the "Number of People Required" for now, this problem is simple, and thus the earliest possible completion time can be easily computed. It is 9 weeks.* Figure 12.5 shows a proposed activity schedule that will achieve this overall completion time. Thus, Figure 12.5 respects the precedence relationships of Figure 12.4, and at the same time shows when each

FIGURE 12.4

Precedence Relationships

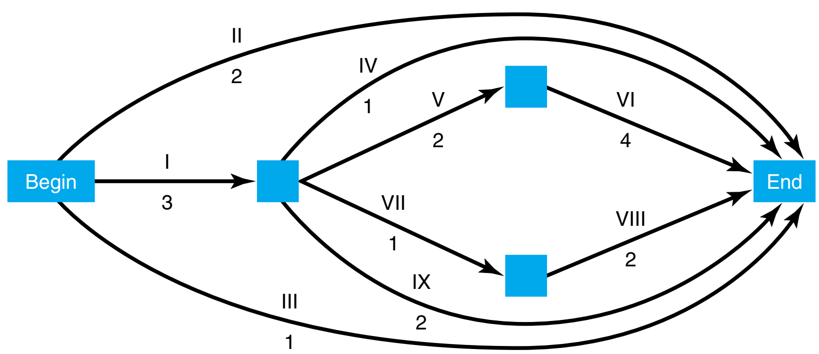


Table 12.2

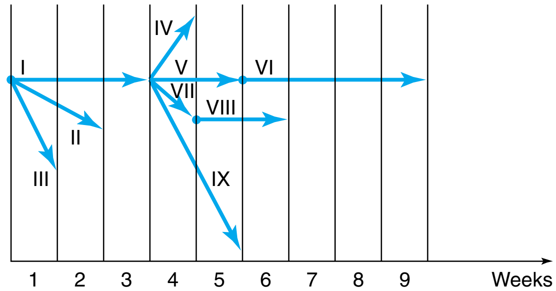
Requirements for Each Activity

ACTIVITY	TIME REQUIRED TO COMPLETE (WEEKS)	NO. OF PEOPLE (PER WEEK) REQUIRED TO COMPLETE
I	3	6
II	2	3
III	1	3
IV	1	3
V	2	6
VI	4	5
VII	1	3
VIII	2	4
IX	2	3

* If you have studied PERT, you can see that activities I, V, and VI form the *critical* path (see Section 14.3).

FIGURE 12.5

Proposed Schedule of Activities



activity should start and how long (in weeks) it will take. In this proposed schedule, each activity starts as early as possible. You can see that I, II, and III start immediately (at the beginning of week 1). Activities IV, V, VII, and IX start at the beginning of week 4. Activity VI starts at the beginning of week 6, and activity VIII starts at the beginning of week 5.

Now consider the number of people per week required to implement the proposed schedule. The personnel data in Table 12.2 can be combined with the schedule in Figure 12.5 to produce the **personnel loading chart** shown in Figure 12.6. As you can see, the proposed schedule makes an erratic utilization of personnel, the requirements fluctuating between the extremes of 15 people in week 4 and only 5 in weeks 7, 8, and 9. It may be to management's advantage to have a schedule that employs resources more smoothly. Heuristic programs are often applied to accomplish such an objective.

WORKLOAD SMOOTHING HEURISTIC

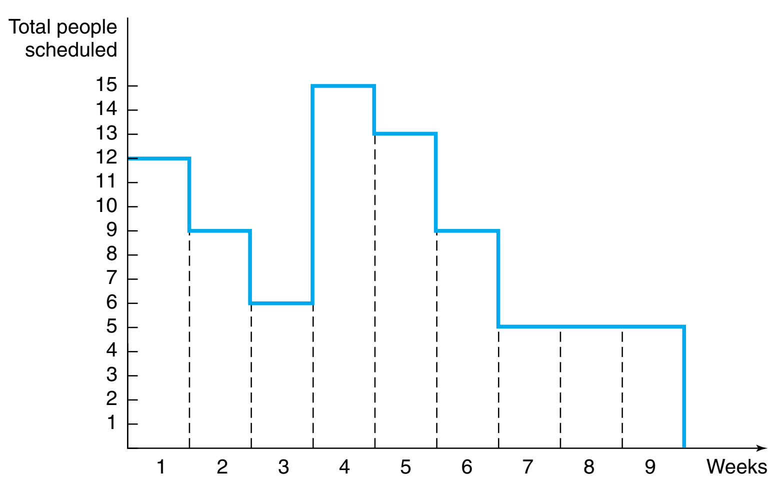
In order to discuss one such heuristic, let us define, for each activity, its **slack**.

> Slack is the maximum amount of time an activity can be delayed without delaying overall project completion.

Note in Figure 12.5 that if the completion time of activity V were delayed, then activity VI could not start at the beginning of week 6 and the project could not be completed by the end of the ninth week. Thus activity V has no slack. In contrast, the completion of activity VIII could be delayed by 3 weeks without delaying the completion of the project. Activity VIII thus has a slack of 3 weeks.

FIGURE 12.6

Personnel Loading Chart for the Proposed Schedule



Now, using this concept, the following heuristic is given to help "reduce the peaks" and "raise the valleys" in order to provide a smoother workload across time:

1. Determine the maximum required resources in the proposed schedule, say *m* workers/week.
2. In each week, impose a new upper limit of $m - 1$ for resource utilization (remember we're trying to reduce the peaks one step at a time), and, if possible, revise the proposed schedule to satisfy this constraint. The revision is systematically performed as follows:
   - **a.** Beginning with the earliest week violating the constraint, consider the activities contributing to the overload and move forward the one with *most* slack as little as possible until it contributes to no overloading, but without delaying the completion of the entire project (which means that activities with zero slack may not be moved). If there are ties, move forward the activity that contributes *least* to the overload (i.e., requires the fewest people).
   - **b.** The heuristic terminates when the current overload cannot be decreased.

To apply this heuristic, let us portray the proposed plan as in Figure 12.7. In this figure, the activity label (e.g., I, II, etc.) appears *below* each arrow. *Above* each arrow is the number of people required each week. For example, the 6 above activity I implies that 6 people are required for each of the 3 weeks needed to complete activity I. Thus, you can read down the appropriate columns to obtain total personnel utilization in a given week. For example, since week *2* is intersected by activities I and II, the entry in the Total Personnel row, under the week 2 column, is 9. Similarly, the distance from the head of each unfollowed arrow at the end of a series of jobs to the end of week 9 indicates the slack for such an arrow. Thus, activity IV has 5 weeks of slack, while activity VIII has 3 weeks of slack, and so on. For activity VII, which is a followed arrow, we compute the slack by noting that VII is followed only by VIII. Since the slack on VIII is 3 weeks, slack on activity VII must also be 3 weeks. Also notice that activities I, V, and VI have zero slack since they cannot be moved forward at all without increasing the overall completion time of 9 weeks. In applying the foregoing heuristic we move forward only activities with positive slack, and hence activities I, V, and VI are not considered.

Applying the Heuristic Given these observations, we may now employ the heuristic. For the first proposal (see Figure 12.7), the maximum required resource is 15 in period 4.

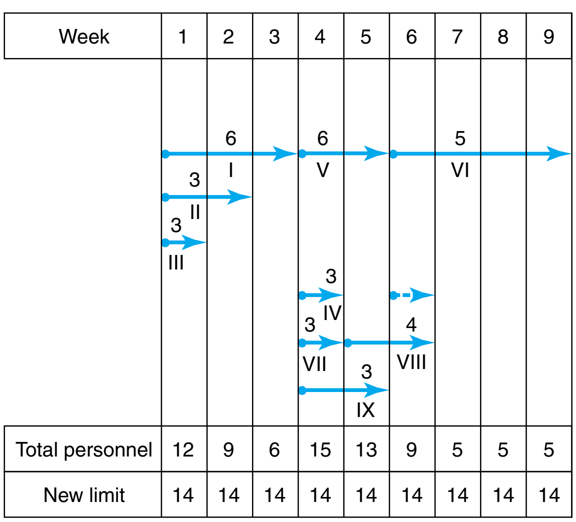


FIGURE 12.7

First Proposal

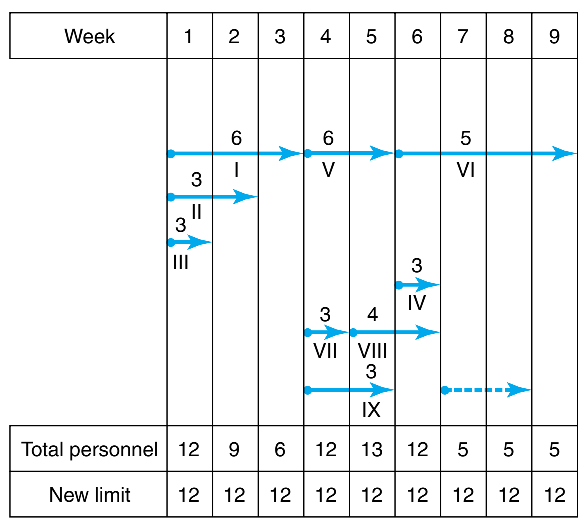


FIGURE 12.8

Second Proposal

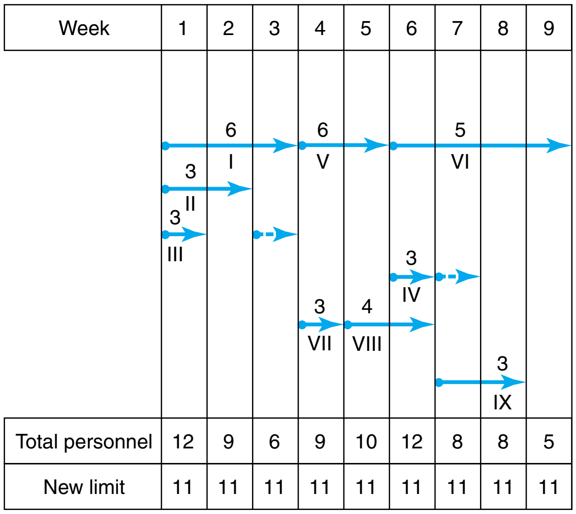


FIGURE 12.9

Third Proposal

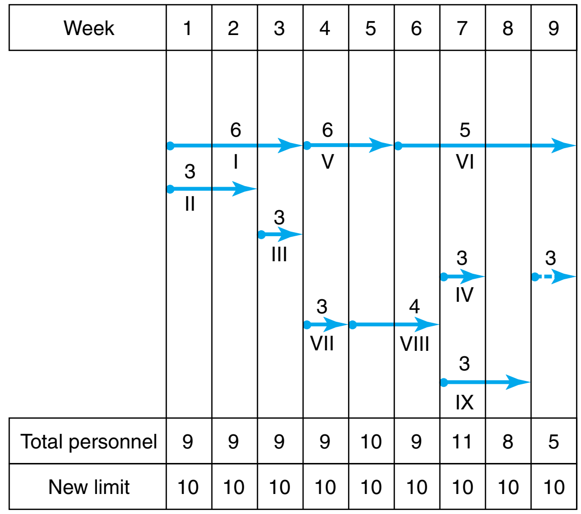


FIGURE 12.10

Fourth Proposal

Thus, according to step 2, we impose a new upper limit of 14 in each week. This limit is violated only in week 4. The "movable" activities contributing to the overload are IV, VII, and IX (since V need not be considered). Of these, the one with most slack is IV. Moving IV forward 1 period reduces the utilization in week 4 by 3 units to 12 people, but creates a utilization of 3 additional units in week 5, giving a total of 16 in week 5, which overloads week 5 (i.e., violates the imposed upper limit of 14). Hence, it must be moved farther forward. You can see that by moving activity IV forward a total of 2 weeks (into week 6 as illustrated in Figure 12.7) no upper limit will be violated. This gives the second proposal, as shown in Figure 12.8.

In this figure the upper limit of 13 must be reduced to 12. The only overload is caused by VIII and IX in week 5. Activity IX has the most slack, and it must be advanced 3 weeks to begin in week 7, as shown. This gives the third proposal presented in Figure 12.9. Here the upper limit of 12 must be reduced to 11. There are violations in weeks 1 and 6. According to the algorithm, we first move III forward 2 weeks and then IV forward 1 week. Continuing with the heuristic, we obtain the fourth and fifth proposals shown in Figures 12.10 and 12.11.

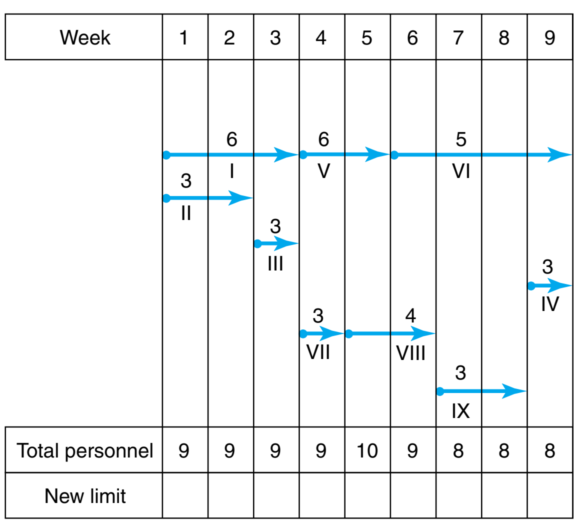
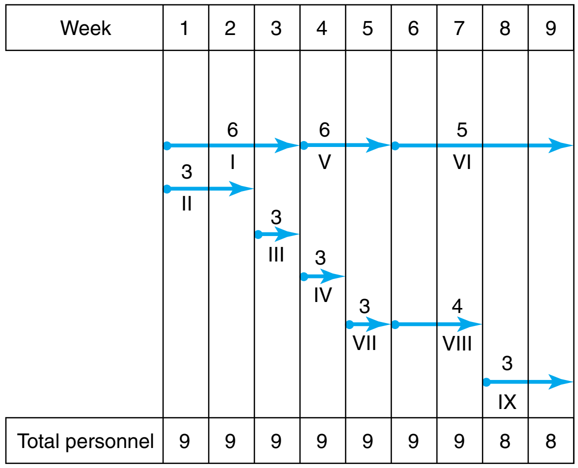


FIGURE 12.11

Fifth Proposal

FIGURE 12.12

Optimal Minimax Schedule



Heuristic Terminates The algorithm is unable to improve beyond the fifth proposal. To see this, note that the overload on week 5 can be reduced only by moving forward activity VIII. However, advancing VIII by 1, 2, or 3 weeks would increase the total personnel in weeks 7 and 8, or 8 and 9, to 12. Step 2b of the heuristic *is* thus satisfied, and hence this schedule is the heuristic solution. This final schedule has smoothed the utilization considerably from that shown in Figure 12.6, for the maximum utilization is now 10 (in week 5), and the minimum is 8.

For this model one might define an optimal solution to be a schedule that *minimizes the maximum utilization of* personnel. An optimal schedule, according to this *minimax* criterion, is shown in Figure 12.12. For this schedule the maximum utilization is 9. Although the heuristic algorithm did not lead to optimality (in this minimax sense—and it must be admitted that the schedule in Figure 12.12 is smoother than that in Figure 12.11), our heuristic approach did quite well. In large models (i.e., with many activities) it would not be possible to easily generate the optimal minimax schedule. It is for this reason that a heuristic is often employed to smooth requirements.

This section and the preceding one have given only a very brief introduction to the important topic of heuristic algorithms. Another example, assigning facilities to different locations within the building (sometimes called the "layout" model), is discussed in Problems 12-2, 12-3, and 12-4 at the end of this chapter.

12.4 MULTIPLE OBJECTIVES

In many applications, the planner has more than one objective. These different objectives may all be of equal importance or, at the very least, it may be difficult for the planner to compare the importance of one objective with that of another. The presence of multiple objectives is frequently referred to as the problem of "combining apples and oranges." Consider for example the corporate planner whose long-range goals are to (1) maximize discounted profits, (2) maximize market share at the end of the planning period, and (3) maximize existing physical capital at the end of the planning period. These goals are not commensurate, which means that they cannot be *directly* combined or compared. It is also clear that the goals are *conflicting.* That is, there are trade-offs in the sense that sacrificing the requirements on any one goal will tend to produce greater returns on the others. For example, spending fewer dollars on marketing is apt to reduce market share and thus prevent the firm from meeting its second goal. However, these dollars can be spent on new machinery in an effort to increase physical capital and satisfy the third goal.

The treatment of multiple objectives is a young but important area in application. At this time the analytic methods for handling models with multiple objectives have not been applied as often in practice as some of the other models, such as linear programming, forecasting, inventory control, and Monte Carlo simulation. However, the concepts involved are important, and some leaders in the management science community feel that they will become more important in the near future. The models have been found to be especially useful on problems in the public sector.

Several approaches to multiple objective models (also called multi-criteria decision making) have been developed. They are: use of multi-attribute utility theory, search for Pareto optimal solutions via multi-criteria linear programming, analytic hierarchy process (AHP), and goal programming. Our discussion is limited to the last two: AHP and goal programming. AHP was developed by Thomas Saaty [see Saaty] and is a relatively new approach to help managers choose between many decision alternatives on the basis of multiple criteria. Goal programming (GP) was a concept introduced by A. Charnes and W. W. Cooper [see Charnes and Cooper] which in some ways can be thought of as a heuristic approach to the multiple-objectives model. GP is a powerful approach that builds on the development of linear programming presented in Chapters 3 and 4. Both areas are now experiencing considerable interest and development and are potentially important topics for future managers.

APPLICATION CAPSULE Facilities Planning at the University of Missouri

In the academic environment, the thrust for excellence in teaching, research, and extension service presents peculiar challenges for the facilities planner who must try to allocate limited resources to achieve the optimal trade-off among the often-conflicting university objectives. The department of engineering management at the University of Missouri-Rolla benefited from a major expansion of its physical facilities in 1987. This included 5,072 square feet of floor space to be developed into a computer integrated manufacturing (CIM) laboratory for teaching, research, and extension. The facility was intended to stimulate interest in teaching and research in advance manufacturing systems and was expected to evolve into a center for technical excellence for industry in the state.

The university saw the new CIM lab facility as a campus-wide resource to be used by all departments in the school of engineering and by various research centers within the university. This resulted in contentious debate and discussion concerning the best facility layout for the new lab. A task force was appointed to resolve the conflict. After identifying alternative layout proposals, 15 sections were identified to be located in the CIM lab (e.g., physical simulation area, Autocad, robot system, etc.). The ideal area for each section was estimated. The sum of the ideal requirements was 6,035 square feet, nearly a 1000 more than was available.

The team had to find some systematic way to allocate actual space available to the desired sections in a manner consistent with the overall mission of the university. Five goals were established (e.g., develop new courses relying on the lab, heighten industry awareness of CIM concepts, etc.) and analytic hierarchy process (AHP) was used to determine how to prioritize the goals. A questionnaire was administered to the team to elicit relative priorities. The preliminary analysis of the responses revealed several inconsistencies in the subjective pairwise comparison of attributes. The respondents had an opportunity to review and adjust their responses, which resulted in greater consistency in the subjective comparisons.

Once the priorities were established, a linear goal programming model was used to determine the allocation of space to each of the 15 areas. Nine of the 15 obtained space allocation factors less than 1.0, which suggested a reduction in the ideal areas originally allocated. Four areas were reduced significantly. The committee used the space allocation as a guide to developing the initial layout of the lab. Sensitivity analysis was also used to determine the effect of altering the priorities and the model was found to be fairly robust in response to priority rankings.

AHP was found to be an effective methodology to obtain group consensus in a highly political environment in a timely manner for a fairly complex institutional planning problem. Because this systematic planning methodology was adopted, the school of engineering readily accepted the department's proposals for the layout of the CIM lab and acknowledged that future lab development should be entrusted to the engineering management department. The CIM lab has handsomely fulfilled its teaching, research, and extension objectives. (See Benjamin et. al.)

GOAL PROGRAMMING

Goal programming is generally applied to linear models; it is an extension of LP that enables the planner to come as close as possible to satisfying various goals and constraints. It allows the decision-maker, at least in a heuristic sense, to incorporate his or her preference system in dealing with multiple conflicting goals. It is sometimes considered to be an attempt to put into a mathematical programming context the concept of *satisficing.* This term was coined by Herbert Simon, a Nobel Prize winner in economics, to communicate the idea that individuals often do not seek optimal solutions, but rather, they seek solutions that are "good enough" or "close enough," or in other words, the desire to maximize several objectives simultaneously to at least satisfactory levels. We shall illustrate the method of goal programming with several examples.

Suppose that we have an educational program design model with decision variables x_1 and x_2, where x_1 is the hours of classroom work and x_2 is the hours of laboratory work. Assume that we have the following constraint on total program hours:

$$x_1 + x_2 \le 100 \qquad \text{(total program hours)}$$

Two Kinds of Constraints In the goal programming approach there are two kinds of constraints: (1) *system constraints* (so-called hard constraints) that cannot be violated and (2) *goal constraints* (so-called soft constraints) that may be violated if necessary. The above constraint on total program hours is an example of a system constraint.

Now, in the program we are designing, suppose that each hour of classroom work involves 12 minutes of small-group experience and 19 minutes of individual problem solving, whereas each hour of laboratory work involves 29 minutes of small-group experience and 11 minutes of individual problem solving. Note that the total program time is at most 6,000 minutes (100 hr * 60 min/hr). The designers have the following two *goals*: Each student should spend as close as possible to one fourth of the maximum program time working in small groups and one third of the time on problem solving. These conditions are

$$12x_1 + 29x_2 \cong 1500 \qquad \text{(small-group experience)}$$

$$19x_1 + 11x_2 \cong 2000 \qquad \text{(individual problem solving)}$$

where the symbol $\cong$ means that the left-hand side is desired to be "as close as possible" to the RHS. If it were possible to find a policy that exactly satisfies the small-group and problem-solving goals (i.e., exactly achieves both right-hand sides), without violating the system constraint on total program hours, then this policy would solve the model. A simple geometric analysis will show that no such policy exists. Clearly then, in order to satisfy the system constraint, at least one of the two goals will be violated.

To implement the goal programming approach, the small-group experience condition is rewritten as the goal constraint

$$12x_1 + 29x_2 + u_1 - v_1 = 1500 \ (u_1 \ge 0, v_1 \ge 0)$$

where u_1 = the amount by which total small-group experience falls short of 1500

v_1 = the amount by which total small-group experience exceeds 1500

Deviation Variables The variables u_1 and v_1 are called **deviation variables**, since they measure the amount by which the value produced by the solution deviates from the goal. We note that by definition we want either u_1 or v_1 (or both) to be zero because it is impossible to simultaneously exceed and fall short of 1500. In order to make $12x_1 + 29x_2$ as close as possible to 1500, it suffices to make the sum $u_1 + v_1$ small.

In a similar way, the individual problem-solving condition is written as the goal constraint

$$19x_1 + 11x_2 + u_2 - v_2 = 2000 \ (u_2 \ge 0, v_2 \ge 0)$$

and in this case we want the sum of the two deviation variables $u_2 + v_2$ to be small. Our complete (illustrative) model is now written as follows:

$$\begin{aligned}
&\text{Min } u_1 + v_1 + u_2 + v_2\\
&\quad\text{s.t.}\quad x_1 + x_2 \le 100 \text{ (total program hours)}\\
&12x_1 + 29x_2 + u_1 - v_1 = 1500 \text{ (small-group experience)}\\
&19x_1 + 11x_2 + u_2 - v_2 = 2000 \text{ (problem solving)}\\
&\quad x_1, x_2, u_1, v_1, u_2, v_2 \ge 0\\
&\textit{Note: } \text{Both } u_1 \text{ and } v_1 \text{ can't be} > 0.
\end{aligned}$$

This is an ordinary LP model and can now be easily solved in Excel. The optimal decision variables will satisfy the system constraint (total program hours). Also, it turns out that the Solver (for technical reasons that we shall not dwell on) will guarantee that either u_1 or v_1 (or both) will be zero, and thus these variables automatically satisfy this desired condition. The same statement holds for u_2 and v_2 and in general for any pair of deviation variables.

Note that the objective function is the sum of the deviation variables. This choice of an objective function indicates that we have no preference among the various deviations from the stated goals. For example, any of the following three decisions is acceptable: (1) a decision that overachieves the group experience goal by 5 minutes and hits the problem-solving goal exactly, (2) a decision that hits the group experience goal exactly and underachieves the problem-solving goal by 5 minutes, and (3) a decision that underachieves each goal by 2.5 minutes. In other words, we have no preference among the three solutions:

(1)		(2)		(3)	
	$u_1 = 0$		$u_1 = 0$		$u_1 = 2.5$
	$v_1 = 5$		$v_1 = 0$		$v_1 = 0$
	$u_2 = 0$		$u_2 = 5$		$u_2 = 2.5$
	$v_2 = 0$		$v_2 = 0$		$v_2 = 0$

We must have no preference because each of these three decisions yields the same value (i.e., 5) for the objective function.

Weighting the Deviation Variables Such a lack of preference for one solution over another certainly would not hold for all goal programming models. Differences in units alone could produce a preference among the deviation variables. Suppose, for example, that the individual problem-solving constraint had been written in hours; that is,

$$\frac{19}{60}x_1 + \frac{11}{60}x_2 + u_2 - y_2 = \frac{2000}{60}$$

It is hard to believe that the program designers would not prefer a 1-minute excess of small-group experience ($v_1 = 1$) to a 1-hour shortfall of individual problem solving ($u_2 = 1$).

One way of expressing a preference among the various goals is to assign different coefficients to the deviation variables in the objective function. In the program-planning example one might select

$$\text{Min } 10u_1 + 2v_1 + 20u_2 + v_2$$

as the objective function. Since v_2 (overachievement of problem solving) has the smallest coefficient, the program designers would rather have v_2 positive than any of the other deviation variables (positive v_2 is penalized the least). Indeed, with this objective function it is better to be 9 minutes over the problem-solving goal than to underachieve by 1 minute the small-group-experience goal. To see this, note that for any solution in which $u_1 \geq 1$, decreasing u_1 by 1 and increasing v_2 by 9 would yield a smaller value for the objective function.

Goal Interval Constraints Another type of goal constraint is called a **goal interval constraint**. Such a constraint restricts the goal to a range or *interval* rather than a specific numerical value. Suppose, for example, that in the above illustration the designers were indifferent among programs for which

$$1800 \leq [\text{minutes of individual problem solving}] \leq 2100$$

$$\text{i.e., } 1800 \leq 19x_1 + 11x_2 \leq 2100$$

In this situation the interval goal is captured with two goal constraints:

$$19x_1 + 11x_2 - v_1 \leq 2100 \qquad (v_1 \geq 0)$$

$$19x_1 + 11x_2 + u_1 \geq 1800 \qquad (u_1 \geq 0)$$

When the terms u_1 and v_1 are included in the objective function, the LP code will attempt to minimize them. We note that when, at optimality, $u_1^* = 0$ and $v_1^* = 0$ (their minimum possible values), the total minutes of problem solving $(19x_1 + 11x_2)$ fall within the desired range (i.e., $1800 \leq 19x_1 + 11x_2 \leq 2100$). Otherwise it will turn out that, at optimality, one of the two variables will be positive and the other zero, which means that only one side of the two-sided inequality can be satisfied.

Summary of the Use of Goal Constraints It may be useful at this point to summarize the various ways in which goal constraints can be formulated and employed. Each goal constraint consists of a left-hand side, say $g_i(x_1, \ldots, x_n)$, and a right-hand side, b_i. Goal constraints are written by using nonnegative deviation variables u_i, v_i. At optimality at least one of the pair u_i, v_i will always be zero. The variable u_i represents *underachievement*; v_i represents *overachievement*. Whenever u_i is used it is *added* to $g_i(x_1, \ldots, x_n)$. Whenever v_i is used, it is *subtracted* from $g_i(x_1, \ldots, x_n)$. Only deviation variables (or a subset of deviation variables) appear in the objective function, and the objective is always to minimize. The decision variables x_i, $i = 1, \ldots, n$ do not appear in the objective. We have discussed four types of goals:

1. ***Target.*** Make $g_i(x_1, \ldots, x_n)$ as close as possible to b_i. To do this we write the goal constraint as

   $$g_i(x_1, \ldots, x_n) + u_i - v_i = b_i \qquad (u_i \geq 0, v_i \geq 0)$$

   and in the objective we minimize $u_i + v_i$. At optimality, at least one of the variables u_i, v_i will be zero.
2. ***Minimize Underachievement.*** To do this, we can write

   $$g_i(x_1, \ldots, x_n) + u_i - v_i = b_i \qquad (u_i \geq 0, v_i \geq 0)$$

   and in the objective we minimize u_i, the underachievement. Since v_i does not appear in the objective function, and it is only in this constraint, hence the constraint can be equivalently written as

   $$g_i(x_1, \ldots, x_n) + u_i \geq b_i \qquad (u_i \geq 0)$$

   If the optimal u_i is positive, this constraint will be active, for otherwise u_i^* could be made smaller. This result is also clear from the equality form of the constraint. That is, if $u_i^* > 0$ then, since v_1^* must equal zero, it must be true that $g_i(x_1, \ldots, x_n) + u_i^* = b_i$.
3. ***Minimize Overachievement.*** To do this, we can write

   $$g_i(x_1, \ldots, x_n) + u_i - v_i = b_i \qquad (u_i \geq 0, v_i \geq 0)$$

and in the objective we minimize v_i, the overachievement. Since in this case u_i does not appear in the objective function, the constraint can be equivalently written as

$$g_i(x_1, \ldots, x_n) - v_i \le b_i \qquad (v_i \ge 0)$$

If the optimal v_i is positive, this constraint will be active. The argument is analogous to that in item 2 above.

4. ***Goal Interval Constraint.*** In this instance, the goal is to come as close as possible to satisfying

$$a_i \le g_i(x_1, \ldots, x_n) \le b_i$$

In order to write this as a goal, we first "stretch out" the interval by writing

$$a_i - u_i \le g_i(x_1, \ldots, x_n) \le b_i + v_i \qquad (u_i \ge 0, v_i \ge 0)$$

which is equivalent to the two constraints

$$\begin{array}{lll} g_i(x_i, \ldots, x_n) + u_i \ge a_i & g_i(x_i, \ldots, x_n) + u_i - \hat{v}_i = a_i & (u_i \ge 0, \hat{v}_i \ge 0) \\ g_i(x_i, \ldots, x_n) - v_i \le b_i & g_i(x_i, \ldots, x_n) + \hat{u}_i - v_i = b_i & (\hat{u}_i \ge 0, v_i \ge 0) \end{array}$$

In the case of a goal interval constraint we minimize $u_i + v_i$ in the objective function. The variables $\hat{v}_i$ and $\hat{u}_i$ are merely surplus and slack, respectively (not deviation variables). As usual, at optimality, at least one of the deviation variables u_i, v_i will be zero. In dealing with two constraints representing a goal interval, the constraint with the nonzero deviation variable (if there is one) will be active.

In general, goal constraints are most often expressed in the appropriate equality form using deviation variables, surplus, and slack as required. The equivalent inequality forms that we have displayed will allow us, for models in two decision variables, to obtain some geometric insight into the solution procedure.

ABSOLUTE PRIORITIES

In some cases managers do not wish to express their preferences among various goals in terms of weighted deviation variables, for the process of assigning weights may seem too arbitrary or subjective. In such cases it may be more acceptable to state preferences in terms of **absolute priorities** (as opposed to weights) to a set of goals. This approach, which requires that goals be satisfied in a specific order, is illustrated in the following example. With weightings, the goal programming model is solved just once. With priorities, the goal programming model is solved in stages as a sequence of models.

Swenson's Media Selection Model: A Minicase Tom Swenson, a senior partner at J. R. Swenson, his father's advertising agency, has just completed an agreement with a pharmaceutical manufacturer to mount a radio and television campaign to introduce a new product, Mylonal. The total expenditures for the campaign are not to exceed \$120,000. The client is interested in reaching several audiences with this campaign. To determine how well a particular campaign meets this client's needs, the agency estimates the impact of the advertisements on the audiences of interest. The impact is measured in *rated exposures,* a term that means "people reached per month." Radio and television, the two media the agency is considering using, are not equally effective in reaching all audiences. Data relevant to the Mylonal campaign are shown in Table 12.3.

After lengthy discussions with the client, Tom accepts the following goals for this campaign. Tom feels that the order in which he has listed his goals reflects the absolute priority among them.

1. He hopes total exposures will be at least 840,000.
2. In order to maintain effective contact with the leading radio station, he hopes to spend no more than \$90,000 on TV advertising.

Table 12.3

Exposures per $1000 Expenditure

	TV	RADIO
Total	14,000	6000
Upper Income	1200	1200

3. He feels that the campaign should achieve at least 168,000 upper-income exposures.
4. Finally, if all other goals are satisfied, he would like to come as close as possible to maximizing the total number of exposures. He notes that if he spends all of the $120,000 on TV advertising he would obtain 1,680,000 exposures (120 * 14,000), and this is the maximum obtainable.

This is clearly a model with a number of constraints. It is not quite a typical mathematical programming model, however, since Tom has a number of objectives. Nevertheless, he feels that a mathematical programming approach will help him understand and solve the model. He thus proceeds in the typical manner. To model the problem, he introduces the notation

x_1 = dollars spent on TV (in thousands)

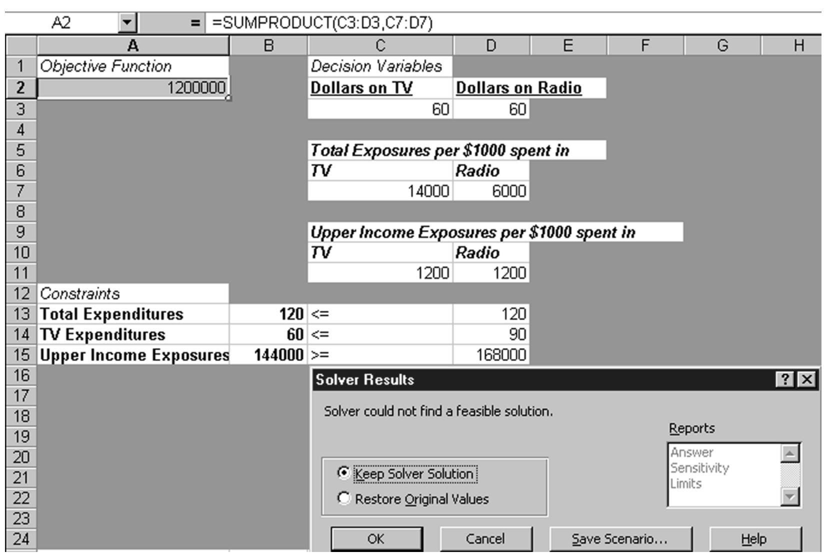
x_2 = dollars spent on radio (in thousands)

Since his highest-priority goal is total exposures, he feels that a reasonable way to model the problem is to use total exposures as the objective function and to treat the other goals as constraints.

An Infeasible Model The formulation and spreadsheet solution of this model ("Base Case" of SWENSON.XLS) are shown in Figure 12.13. Each constraint and the objective function are labeled to indicate the purpose they serve. The Solver Results dialog

FIGURE 12.13

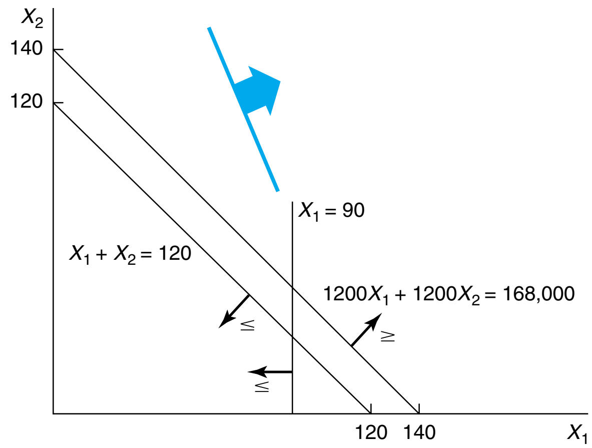
Maximizing Total Exposures



Cell	Formula	Copy To
A2	= SUMPRODUCT(C3:D3,C7:D7)	—
B13	= SUM(C3:D3)	—
B14	= C3	—
B15	= SUMPRODUCT(C3:D3,C11:D11)	—

FIGURE 12.14

Maximizing Total Exposures: A Graphical Approach



box tells Tom that the model is infeasible. Clearly, since it is infeasible, there is no way to satisfy simultaneously the three goals (total expenditures, TV expenditure, and upper-income exposures) that Tom has stated as constraints. Since there are only two decision variables in this model, the graphical approach can be used to investigate Tom's initial formulations. The analysis in Figure 12.14 clearly shows that there are no points that satisfy both the first (total expenditures) and the third (upper-income exposures) constraints. At this point, Tom could attempt to approach the model somewhat differently. He might change one or more of his goals, or perhaps the objective function, and start again. In general, however, this is not a satisfactory systematic approach. In models with many decision variables and several conflicting goals, restructuring the model to create a new model that has a feasible solution could prove to be difficult. More important, in this restructuring process, the essence of the real model could be lost.

Recall that Tom is not indifferent about the various goals; indeed, he has stated an absolute priority among them. Goal programming with absolute priorities is designed to handle exactly the type of decision process Tom Swenson wants. It is a sequential process in which goals are added one at a time (in the order of decreasing priority) to an LP model.

Swenson's Goal Programming Model In order to set up his model as a goal program, Tom notes that the first goal, if violated, will be underachieved. The second goal, if violated, will be overachieved, and so on. Employing this reasoning, he restates his goals, in descending priority, as

1. Minimize the underachievement of 840,000 total exposures (i.e., Min u_1, subject to the condition $14{,}000\ x_1 + 6000x_2 + u_1 \geq 840{,}000$; $u_1 \geq 0$).
2. Minimize expenditures in excess of $90,000 on TV (i.e., Min v_2, subject to the condition $x_1 - v_2 \leq 90$; $v_2 \geq 0$).
3. Minimize underachievement of 168,000 upper-income exposures (i.e., Min u_3, subject to the condition $1200x_1 + 1200x_2 + u_3 \geq 168{,}000$; $u_3 \geq 0$).
4. Minimize underachievement of 1,680,000 total exposures—the maximum possible (i.e., Min u_4, where $14{,}000x_1 + 6000x_2 + u_4 \geq 1{,}680{,}000$; $u_4 \geq 0$).

Note that Tom's priorities are now clearly stated in terms of either minimizing underachievement (i.e., minimizing a u_i) or minimizing overachievement (i.e., minimizing a v_i). His goals, as stated above, have been expressed as inequalities in accord with our previous discussion. This method will facilitate a graphical analysis.

Given that he has correctly formulated his priorities, Tom must distinguish between (1) *system constraints* (all constraints that may not be violated) and (2) *goal constraints.* In

FIGURE 12.15

Goal Program Formulation

$$
\begin{array}{llcccccl}
\text{Min } P_1u_1 + P_2v_2 + P_3u_3 + P_4u_4 \\
\text{s.t.} & x_1 & + & x_2 & & \le & 120 & \text{(S)} \\
& 14{,}000x_1 & + & 6000x_2 & + u_1 & \ge & 840{,}000 & (1) \\
& x_1 & & & - v_2 & \le & 90 & (2) \\
& 1200x_1 & + & 1200x_2 & + u_3 & \ge & 168{,}000 & (3) \\
& 14{,}000x_1 & + & 6000x_2 & + u_4 & \ge & 1{,}680{,}000 & (4) \\
& & & x_1, x_2, u_1, v_2, u_3, u_4 & & \ge & 0
\end{array}
$$

his model, the only system constraint is that total expenditures will be no greater than \$120,000. Thus (since the units of x_1 and x_2 are thousands), we have

$$x_1 + x_2 \le 120 \qquad \text{(S)}$$

In goal programming notation, Tom's model can now be expressed as follows in Figure 12.15:

Note that the objective function consists only of deviation variables and is of the *Min* form. As already stated, all goal programming formulations are minimization models, as the objective is to come as close as possible to the goals. The terms serve merely to indicate priorities, with P_1 denoting highest priority, and so on. What the problem statement above means precisely is

1. Find the set of decision variables that satisfies the system constraint (S) and that also gives the Min possible value to u_1 subject to constraint (1) and $x_1, x_2, u_1 \ge 0$. Call this set of decisions FR I (i.e., "feasible region I"). Considering *only the highest goal*, all of the points in FR I are "optimal" (i.e., the best that Tom can do) and (again considering only the highest goal) he is indifferent as to which of these points he selects.
2. Find the subset of points in FR I that gives the Min possible value to v_2, subject to constraint (2) and $v_2 \ge 0$. Call this subset FR II. Considering only the ordinal ranking of the two highest-priority goals, all of the points in FR II are "optimal," and in terms of these two highest-priority goals Tom is indifferent as to which of these points he selects.
3. Let FR III be the subset of points in FR II that minimize u_3, subject to constraint (3) and $u_3 \ge 0$.
4. FR IV is the subset of points in FR III that minimize u_4, subject to constraint (4) and $u_4 \ge 0$. Any point in FR IV is an optimal solution to Tom's overall model.

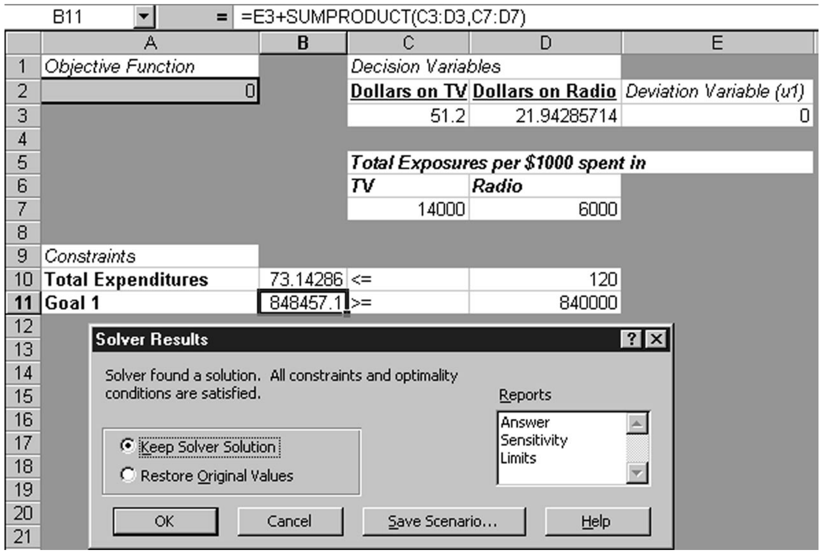
Graphical Analysis and Spreadsheet Implementation of the Solution Procedure Since Tom's marketing model has only two decision variables, the solution method above can be accomplished with graphical analysis. In all real-world models, the spreadsheet with its Solver tool would be used. In the next section we show how this can be done using LP.

1. In Figure 12.16, both the spreadsheet output (new sheet called "First Goal" in the same SWENSON.XLS) and the geometry reveal that the Min of u_1 s.t. (S), (1), and x_1, x_2, $u_1 \ge 0$ is $u_1^* = 0$. Although Solver returns optimal values for x_1^* and x_2^*, these values are not of interest. The important information is that $u_1^* = 0$, which tells us that the first goal can be completely attained. Alternative optima for the current model are provided by all values of (x_1, x_2) that satisfy the conditions

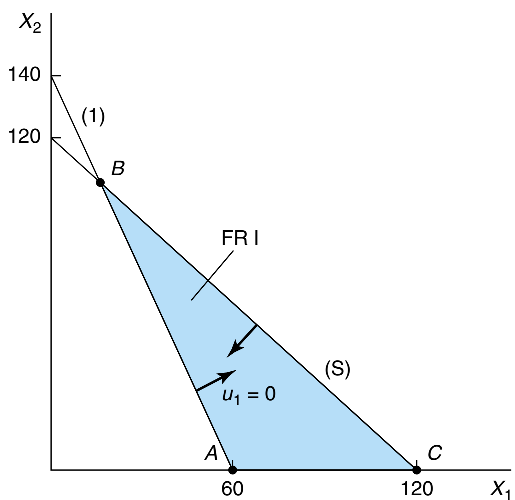
$$\text{FR I}\begin{cases} x_1 + x_2 \le 120 \\ 14{,}000x_1 + 6000x_2 \ge 840{,}000 \\ x_1, x_2 \ge 0 \end{cases}$$

FIGURE 12.16

First Goal



Cell	Formula	Copy To
A2	= E3	—
B10	= SUM(C3:D3)	—
B11	= E3+SUMPRODUCT(C3:D3,C7:D7)	—



At any such point Tom's first goal is attained ($u_1^* = 0$) so that, in terms of only the first goal, these decisions are equally preferable. Thus FR I is the shaded area *ABC* in Figure 12.16.

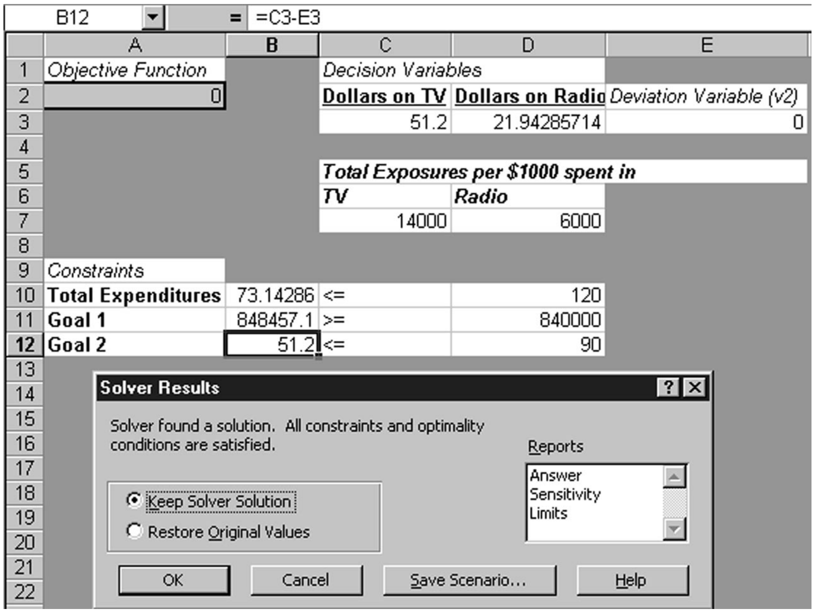
The line labeled (1) represents goal 1. The arrow marked $u_1 = 0$ indicates that at all points to the right of line (1) goal 1 is achieved.

2. In the spreadsheet formulation in Figure 12.17 (new sheet called "Goal 2"), we have entered the constraints defining FR I (constraints in cells B10:D11), together with the new goal constraint (2) (shown in cells B12:D12), and we see that

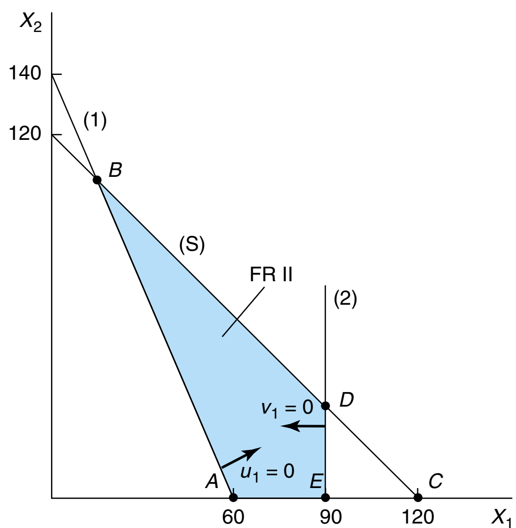
$$\text{Min } v_2$$
$$\text{s.t.} \quad x \text{ in FR I, goal (2), and } v_2 \geq 0$$

FIGURE 12.17

Goal 2



Cell	Formula	Copy To
A2	= E3	—
B10	= SUM(C3:D3)	—
B11	= SUMPRODUCT(C3:D3,C7:D7)	—
B12	= C3–E3	—



is $v_2^* = 0$. Thus, FR II is defined by

$$\text{FR II} \begin{cases} x_1 + x_2 \leq 120 \\ 14{,}000x_1 + 6000x_2 \geq 840{,}000 \\ x_1 \leq 90 \\ x_1, x_2 \geq 0 \end{cases}$$

which is the shaded area *ABDE*, clearly a subset of FR I. As expected, the size of the feasible region has become smaller.

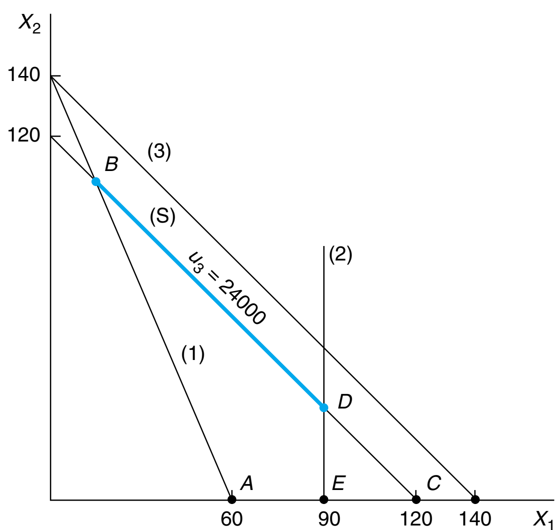
FIGURE 12.18

Goal 3

B16 = =E3+SUMPRODUCT(C3:D3,C11:D11)

	A	B	C	D	E	F	G
1	*Objective Function*		*Decision Variables*				
2	24000		**Dollars on TV**	**Dollars on Radio**	*Deviation Variable (u3)*		
3			81.429	38.571	24000		
4							
5			***Total Exposures per $1000 spent in***				
6			***TV***	***Radio***			
7			14000	6000			
8							
9			***Upper Income Exposures per $1000 spent in***				
10			***TV***	***Radio***			
11			1200	1200			
12	*Constraints*						
13	**Total Expenditures**	120	<=	120			
14	**Goal 1**	1371429	>=	840000			
15	**Goal 2**	81.42857	<=	90			
16	**Goal 3**	168000	>=	168000			

Cell	Formula	Copy To
A2	= E3	—
B13	= SUM(C3:D3)	—
B14	= SUMPRODUCT(C3:D3,C7:D7)	—
B15	= C3	—
B16	= E3+SUMPRODUCT(C3:D3,C11:D11)	—



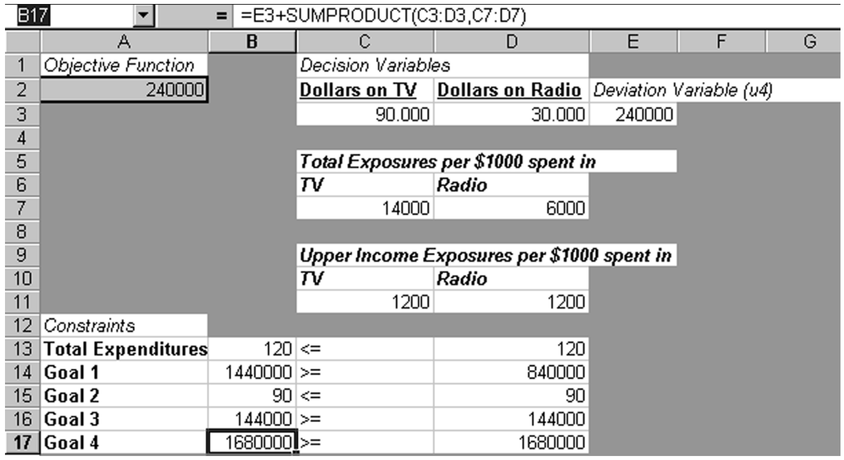
Continuing in this way, Figure 12.18 (new sheet called "Goal 3") shows that FR III is the line segment *BD*. In this case $u_3^* = 24{,}000$. Although the first two goals were completely attained (since $u_1^* = v_2^* = 0$), the third goal cannot be completely attained because $u_3^* > 0$. At this stage, Tom is indifferent about any decision satisfying

$$\text{FR III}\begin{cases} x_1 + x_2 \le 120 \\ 14{,}000x_1 + 6000x_2 \ge 840{,}000 \\ x_1 \le 90 \\ 1200x_1 + 1200x_2 \ge 168{,}000 - 24{,}000 = 144{,}000 \end{cases}$$

which defines the line segment *BD*.

FIGURE 12.19

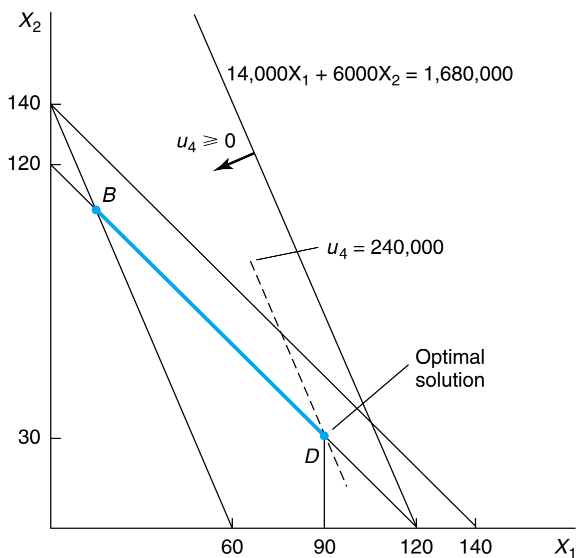
Optimal Solution



B17 = =E3+SUMPRODUCT(C3:D3,C7:D7)

	A	B	C	D	E	F	G
1	*Objective Function*		*Decision Variables*				
2	240000		**Dollars on TV**	**Dollars on Radio**	*Deviation Variable (u4)*		
3			90.000	30.000	240000		
4							
5			***Total Exposures per $1000 spent in***				
6			***TV***	***Radio***			
7			14000	6000			
8							
9			***Upper Income Exposures per $1000 spent in***				
10			***TV***	***Radio***			
11			1200	1200			
12	*Constraints*						
13	**Total Expenditures**	120	<=	120			
14	**Goal 1**	1440000	>=	840000			
15	**Goal 2**	90	<=	90			
16	**Goal 3**	144000	>=	144000			
17	**Goal 4**	1680000	>=	1680000			

Cell	Formula	Copy To
A2	= E3	—
B13	= SUM(C3:D3)	—
B14	= SUMPRODUCT(C3:D3,C7:D7)	—
B15	= C3	—
B16	= SUMPRODUCT(C3:D3,C11:D11)	—
B17	= E3+SUMPRODUCT(C3:D3,C7:D7)	—



Finally, Figure 12.19 (new sheet called "Optimal") shows the optimal solution at point *D*. Recall that the fourth goal is to minimize underachievement of the maximum possible number of exposures, which is 1,680,000. Thus, we wish to minimize the underachievement u_4 where

$$14{,}000x_1 + 6000x_2 + u_4 \geq 1{,}680{,}000.$$

In Figure 12.19 we find the unique optimum $x_1^* = 90$ and $x_2^* = 30$; that is, Tom should spend \$90,000 on TV advertising and \$30,000 on radio advertising. This fact is verified in the geometric analysis, where it is clear that point D ($x_1 = 90$, $x_2 = 30$) is closer to the line that describes goal 4 ($14{,}000x_1 + 6000x_2 = 1{,}680{,}000$) than any other point in FR III (i.e., than any other point on the segment BD). We also note that $u_4^* = 240{,}000$. Thus, Tom achieves only 1,680,000 – 240,000 = 1,440,000 exposures.

We see, then, that goal programming with absolute priorities allows a manager (like Tom) to solve a model in which there is no solution that achieves all the goals, but where he is willing to specify an absolute ranking among the goals and successively restrict his attention to those points that come as close as possible to each goal.

COMBINING WEIGHTS AND ABSOLUTE PRIORITIES

It is possible to combine, to some extent, the concepts of weighted and absolute priority goals. To illustrate this fact, we return to Tom Swenson's advertising model.

In reviewing the results of the absolute priority study, Tom and his client begin to discuss the importance of the older members of the Mylonal market. In particular, they focus on the number of exposures to individuals 50 years old or older. Again, they see that radio and TV are not equally effective in generating exposures in this segment of the population. The exposures per $1000 of advertising are as follows in Table 12.4:

Table 12.4

EXPOSURE GROUP	TV	RADIO
50 and over	3000	8000

If there were no other considerations, Tom would like as many 50-and-over exposures as possible. Since radio yields such exposures at a higher rate than TV (8000 > 3000), Tom sees that the maximum possible number of 50-and-over exposures would be achieved by allocating all of the $120,000 available to radio. Thus, the maximum number of 50-and-over exposures is 960,000 ($=120 \times 8000$). Tom and his client would like to come as close as possible to this goal (minimize underachievement) once the first three goals are satisfied. Recall, however, that they also want to come as close as possible to the goal of 1,680,000 total exposures (minimize underachievement) once the first three goals are satisfied. To resolve this conflict of goals, they decide to use a weighted sum of the deviation variables as the objective in the final phase of the absolute priorities approach. It is their judgment that underachievement in the fifth goal (960,000 exposures to the 50-and-over group) is three times as serious as underachievement in the fourth goal (1,680,000 total exposures). The formulation, its solution (a new sheet called "Weighted" in the same SWENSON.XLS), and graphical analysis are presented in Figure 12.20.

From the spreadsheet we see that the optimal solution to this model is point B ($x_1^* = 15$, $x_2^* = 105$). Recall that when the objective function was to minimize u_4, the optimal decision was point D ($x_1^* = 90$, $x_2^* = 30$). Thus, in the graphical analysis, we see that the new objective function has moved the optimal solution from one end of FR III to the other. There is no obvious graphical way to find the optimal solution to this model; that is, there is not an obvious objective function contour to push in a downhill direction that takes us to the point $x_1 = 15$, $x_2 = 105$. It is, however, intuitively appealing to see that the optimal solution is as close as possible to the more heavily weighted goal. We could perform a sensitivity analysis on the weights in the objective function to see when the solution changes to point B from point D.

This completes the analysis of Tom Swenson's advertising campaign model. The general sequential LP procedure described above for goal programming with absolute priorities holds for any model in which the system constraints and the goal constraints are formulated with linear functions. For each new model a single constraint is added to the previous model, and the objective function is modified slightly. Generally speaking, a fairly

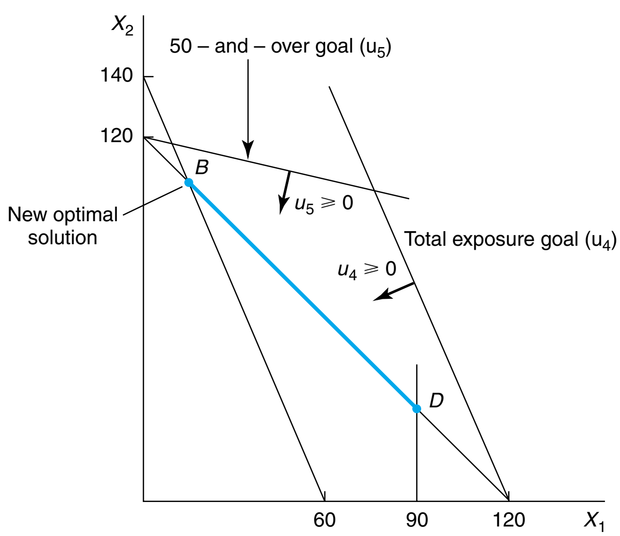
FIGURE 12.20

Weighting the Final Step

A2 = =3*F3+E3

	A	B	C	D	E	F
1	*Objective Function*		*Decision Variables*			
2	1065000		**Dollars on TV**	**Dollars on Radi**	*Deviation Variable (u4)*	*Deviation Variable (u5)*
3			15.000	105.000	840000	75000
4						
5			***Total Exposures per $1000 spent in***			
6			***TV***	***Radio***		
7			14000	6000		
8						
9			***Upper Income Exposures per $1000 spent in***			
10			***TV***	***Radio***		
11			1200	1200		
12			***Senior Exposures per $1000 spent in***			
13			***TV***	***Radio***		
14			3000	8000		
15	*Constraints*					
16	**Total Expenditures**	120	<=	120		
17	**Goal 1**	840000	>=	840000		
18	**Goal 2**	15	<=	90		
19	**Goal 3**	144000	>=	144000		
20	**Goal 4**	1680000	>=	1680000		
21	**Goal 5**	960000	>=	960000		

Cell	Formula	Copy To
A2	= 3*F3+E3	—
B16	= SUM(C3:D3)	—
B17	= SUMPRODUCT(C3:D3,C7:D7)	—
B18	= C3	—
B19	= SUMPRODUCT(C3:D3,C11:D11)	—
B20	= E3+SUMPRODUCT(C3:D3,C7:D7)	—
B21	= F3+SUMPRODUCT(C3:D3,C14:D14)	—



large number of decision variables can be involved. The example with two variables was useful because it made it possible to present, along with the spreadsheet results, geometric interpretations, which add insight to the solution technique.

The foregoing model is useful in indicating how conflicting and noncommensurate goals (i.e., apples and oranges) can be simultaneously considered by means of goal programming. Thus, it gives some insight into why goal programming is a promising and increasingly useful tool in analyzing public policy questions.

12.5 ANALYTIC HIERARCHY PROCESS

This section deals with the real-world topic of making a decision when there are multiple objectives or criteria to consider. There are numerous examples where these kinds of decisions are made every day. Consider the following:

- Choosing which employment offer to accept from among several offers
- Picking which computer to buy (or which automobile, etc.)
- Deciding which new product to launch first
- Selecting a site for a new restaurant, hotel, manufacturing facility, and so on
- Selecting which university to attend
- Identifying the best business or engineering school in the country
- Rating the best cities to live in
- Choosing a new information system for your company that does payroll, accounting, and so forth (or choosing any new software package from competing vendors)
- Selecting what combination of taxes (property, sales, gas, etc.) to levy on the citizens of a locale.

For example, when you go to buy a car, you might consider numerous factors, not the least of which include the price, its safety, the engine size, fuel economy, and so forth. Each of the examples identified previously likewise would have numerous factors to consider in making these complex decisions.

A simple way to attack such a decision would be to assign weights to each of the criteria that were to be considered in making the decision. Then rank each decision alternative on a scale from 1 (worst) to 10 (best). Finally, you would multiply the weights times the rankings for each criterion and sum them up. The alternative with the highest score would be the most preferred. Let's consider an example.

Your boss has asked you to help her buy the next computer for the office. You have to choose between three computers: (1) Model A that runs an AMD K6-II chip at 400 MHz, (2) Model B that runs a Celeron chip at 333 MHz, and (3) Model C that runs a Pentium II chip at 450 MHz. The criteria that are important to you and your boss are price, speed, hard-disk size, and warranty/support. You decide that price should get 50% of the total weight in making the decision, speed 15%, hard-disk size 20%, and warranty/support 15%. Next you rank each of the three models on these four criteria. You rank them on a scale from 1 to 10 (as described earlier) as shown in the following spreadsheet (COMPUTER.XLS) in Figure 12.21.

FIGURE 12.21

Multi-Attribute Decision Model for Buying a Computer

E8 = =SUMPRODUCT($C$4:$C$7,E4:E7)

	A	B	C	D	E	F	G	H
1								
2						Alternatives' Rankings		
3		Criterion	Weights		Model A	Model B	Model C	
4		Price	50%		5	8	3	
5		Speed	15%		7	5	9	
6		Hard-Disk	20%		9	4	10	
7		Warranty	15%		7	10	7	
8			100%		6.4	7.05	5.9	
9								

Cell	Formula	Copy To
C8	= SUM (C4:C7)	—
E8	= SUMPRODUCT ($C$4:$C$7, E4:E7)	F8:G8

As can be seen, Model B comes out with the highest weighted score (7.05) and thus would be your recommendation to your boss. This approach is quite simplistic and there are difficulties in setting the ranking scales on such different criteria.

Analytic hierarchy process (AHP) also uses a weighted average approach idea, but it uses a method for assigning ratings (or rankings) and weights that is considered more reliable and consistent than the simple method described above. AHP is based on pairwise comparisons between the decision alternatives on each of the criteria. Then a similar set of comparisons are made to determine the relative importance of each criterion and thus produces the weights. The basic procedure is as follows:

1. Develop the ratings for each decision alternative for each criterion by
   - developing a pairwise comparison matrix for each criterion
   - normalizing the resulting matrix
   - averaging the values in each row to get the corresponding rating
   - calculating and checking the consistency ratio

2. Develop the weights for the criteria by
   - developing a pairwise comparison matrix for each criterion
   - normalizing the resulting matrix
   - averaging the values in each row to get the appropriate weights
   - calculating and checking the consistency ratio

3. Calculate the weighted average rating for each decision alternative. Choose the one with the highest score.

We will demonstrate this procedure on a new example. Sleepwell Hotels is looking for some help in selecting the "best" revenue management software package from among several vendors. Mark James is the director of revenue management for this chain of hotels and has been given the task of selecting the software package. He has identified three vendors whose software seems to meet their basic needs—Revenue Technology Corporation (RTC), PRAISE Strategic Solutions (PSS), and El Cheapo (EC). The criteria that he thinks are important in making this decision are (1) the total cost of the installed system, (2) the follow-up service provided over the coming year, (3) the sophistication of the underlying math engines, and (4) the amount of customization for Sleepwell. The first step in the AHP procedure is to make pairwise comparisons between the vendors for each criterion. The scale that is used in making these comparisons is a standard one and is described as follows:

RATING	DESCRIPTION
1	Equally preferred
3	Moderately preferred
5	Strongly preferred
7	Very strongly preferred
9	Extremely strongly preferred

Values of 2, 4, 6, or 8 may also be assigned and represent preferences halfway between the integers on either side (e.g., a 2 is between a 1 and a 3—somewhere between equally preferred and moderately preferred).

Mark starts with the first criterion (total cost) and generates the following data in his spreadsheet (sheet "Total Cost" in SLEEPWLL.XLS shown in Figure 12.22). The table is read as follows: The vendor in the row is being compared to the vendor in the column. If the vendor in the row is preferred to the vendor in the column, then a number from 1 to 9 (from the AHP table) is assigned to the cell at the intersection of the row and column. If, however, the vendor in the column is preferred to the vendor in the row, then 1 divided by

FIGURE 12.22

Pairwise Comparison on Total Cost

B6 = =1/D4

	A	B	C	D
1				
2				
3		RTC	PSS	EC
4	RTC	1	4	0.5
5	PSS	0.25	1	0.142857
6	EC	2	7	1

Cell	Formula	Copy To
B5	= 1/C4	—
B6	= 1/D4	—
C6	= 1/D5	—

(a number from 1 to 9) is assigned to the cell at the intersection of the row and column. Obviously since vendor 1 (RTC) is equally preferred to vendor 1, then a "1" is assigned to that row/column and in fact, all along the diagonal. Vendor 1 is moderately to strongly preferred to vendor 2 on the Total Cost basis and so a "4" is assigned in the first row, second column (cell C4). Vendor 3 (EC) is equally to moderately preferred to Vendor 1 (RTC) and so a "1/2" is assigned in row 1, column 3 (cell D4). Mark has set up his spreadsheet so that once the entries above and to the right of the diagonal are entered (cells C4, D4, and D5), it automatically calculates the reciprocal preferences. For example, since vendor 1 compared to vendor 2 was assigned a "4," then the comparison of vendor 2 to vendor 1 gets a "1/4" automatically (cell B5).

Once all the relevant pairwise comparisons have been made, the matrix needs to be normalized. This is done by totaling the numbers in each column. Each entry in the column is then divided by the column sum to yield its normalized score. Mark has now done this in his spreadsheet and it is shown in cells B12:D14 of Figure 12.23. His next step is to calculate the average score of each vendor for the "Total Cost" criterion. These values are shown in column E of Figure 12.23. Mark sees that EC has the highest average score on this factor.

Once the normalized matrix is finished, he must calculate the consistency ratio and check its value. The purpose for doing this is to make sure Mark was consistent in the preference ratings he expressed in the original table. For example, if Mark expressed strong

FIGURE 12.23

Normalized Matrix for Total Cost

B12 = =B4/B$8

	A	B	C	D	E
1					
2					
3		RTC	PSS	EC	
4	RTC	1	4	0.5	
5	PSS	0.25	1	0.142857	
6	EC	2	7	1	
7					
8	Sum	3.25	12	1.642857	
9					
10	*NORMALIZED*				
11		RTC	PSS	EC	Average
12	RTC	0.308	0.333	0.304	0.315
13	PSS	0.077	0.083	0.087	0.082
14	EC	0.615	0.583	0.609	0.602

Cell	Formula	Copy To
B8	= SUM(B4:B6)	C8:D8
B12	= B4/B$8	B12:D14
E12	= AVERAGE(B12:D12)	E13:E14

preference for vendor 1 over vendor 2 on the Total Cost criterion, and moderate preference for vendor 2 over vendor 3, then it would be inconsistent to express equal preference between vendors 1 and 3 or worse to express a preference for vendor 3 over vendor 1. There are three steps in order to arrive at the consistency ratio:

1. Calculate the consistency measure for each vendor.
2. Calculate the consistency index (CI).
3. Calculate the consistency ratio (CI/RI where RI is a random index).

To calculate the consistency measure we can take advantage of Excel's matrix multiplication function =MMULT( ). As Mark shows us in Figure 12.24 that for vendor 1 (RTC) you multiply the average rating for each vendor (cells E12:E14) times the scores in the first row (cells B4:D4) one-at-a-time, sum these products up and divide this sum by the average rating for the first vendor (cell E12). A similar calculation is done for the second and third vendors. Ideally the consistency measures would be equal to the number of decision alternatives in the example (in our case, we have three vendors). To calculate the consistency index (CI), Mark takes the average consistency measure of the three vendors, subtracts the number of alternatives (n), and divides the whole quantity by $n - 1$. This is shown in cell F16 of Figure 12.24 and Mark sees that his CI has a value of 0.001. The final step to find the consistency ratio (CR) is to divide the CI by a random index (RI) that is provided by AHP and shown below:

n	RANDOM INDEX
2	0.00
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.51

This consistency ratio is shown in cell F20 of Figure 12.24 and in Mark's example it equals 0.002.

FIGURE 12.24

Consistency Ratio for Total Cost

F12 = =MMULT(B4:D4,$E$12:$E$14)/E12

	A	B	C	D	E	F
1						
2						
3		RTC	PSS	EC		
4	RTC	1	4	0.5		
5	PSS	0.25	1	0.142857		
6	EC	2	7	1		
7						
8	Sum	3.25	12	1.642857		
9						
10	*NORMALIZED*					
11		RTC	PSS	EC	Average	Consistency Measure
12	RTC	0.308	0.333	0.304	0.315	3.0019
13	PSS	0.077	0.083	0.087	0.082	3.000
14	EC	0.615	0.583	0.609	0.602	3.004
15						
16					CI =	0.001
17						
18					RI=	0.58
19						
20					C. Ratio=	0.002

Cell	Formula	Copy To
F12	= MMULT(B4:D4,$E$12:$E$14)/E12	F13:F14
F16	= (AVERAGE(F12:F14)-3)/2	—
F20	= F16/F18	—

FIGURE 12.25

Consistency Ratio for Service

F20 = =F16/F18

	A	B	C	D	E	F
1						
2						
3		RTC	PSS	EC		
4	RTC	1	0.5	6		
5	PSS	2	1	8		
6	EC	0.166667	0.125	1		
7						
8	Sum	3.166667	1.625	15		
9						
10	NORMALIZED					
11		RTC	PSS	EC	Average	Consistency Measure
12	RTC	0.316	0.308	0.400	0.341	3.0200
13	PSS	0.632	0.615	0.533	0.593	3.0315
14	EC	0.053	0.077	0.067	0.065	3.0034
15						
16					CI =	0.009
17						
18					RI=	0.58
19						
20					C. Ratio=	0.016

For a perfectly consistent manager, the consistency measures will equal *n* and therefore, the CIs will be equal to zero and so will the consistency ratio. If this ratio is very large (Saaty suggests > 0.10), then the manager is not consistent enough and the best thing to do is go back and revise the comparisons (in most cases, you'll have made a simple mistake and this calculation will alert you to that fact).

Mark must now do the same thing for the other three criteria. He can easily do this by copying the "Total Cost" sheet into three other sheets ("Service," "Sophistication," "Custom") and then simply changing the pairwise comparisons. The results of this are shown in Figures 12.25 to 12.27. Mark notes that in all three cases, the CR values range from 0.0 to 0.047, which means he's being consistent. He also notes that PSS is the winner on the Service criterion, RTC and PSS are tied for the best in terms of Sophistication, and PSS is considered the best on Customization.

All of this work concludes the first step in the procedure. The next step (2) in the procedure is to use similar pairwise comparisons to determine the appropriate weights for each of the criteria. The process is the same in that we make comparisons, except that now we make the comparisons between the criteria not the vendors as we did in step 1. Mark does this in a new sheet called "Weights" (in the same workbook) and it's shown in Figure 12.28.

FIGURE 12.26

Consistency Ratio for Sophistication

F16 = =(AVERAGE(F12:F14) - 3)/2

	A	B	C	D	E	F
1						
2						
3		RTC	PSS	EC		
4	RTC	1	1	5		
5	PSS	1	1	5		
6	EC	0.2	0.2	1		
7						
8	Sum	2.2	2.2	11		
9						
10	NORMALIZED					
11		RTC	PSS	EC	Average	Consistency Measure
12	RTC	0.455	0.455	0.455	0.455	3.0000
13	PSS	0.455	0.455	0.455	0.455	3.000
14	EC	0.091	0.091	0.091	0.091	3.000
15						
16					CI =	0.000
17						
18					RI=	0.58
19						
20					C. Ratio=	0.000

FIGURE 12.27

Consistency Ratio for Customization

C8 = =SUM(C4:C6)

	A	B	C	D	E	F	G
1							
2							
3		RTC	PSS	EC			
4	RTC	1	0.25	3			
5	PSS	4	1	6			
6	EC	0.333333	0.166667	1			
7							
8	Sum	5.333333	1.416667	10			
9							
10	*NORMALIZED*						
11		RTC	PSS	EC	Average	Consistency Measure	
12	RTC	0.188	0.176	0.300	0.221	3.0399	
13	PSS	0.750	0.706	0.600	0.685	3.1094	
14	EC	0.063	0.118	0.100	0.093	3.0131	
15							
16					CI =	0.027	
17							
18					RI=	0.58	
19							
20					C. Ratio=	0.047	

FIGURE 12.28

Consistency Ratio for Weights on Criterion

G16 = =(AVERAGE(G12:G15) - 4)/3

	A	B	C	D	E	F	G	H
1								
2								
3		Cost	Service	Sophistication	Customization			
4	Cost	1	6	0.5	3			
5	Service	0.166667	1	0.125	0.333333333			
6	Sophistication	2	8	1	5			
7	Customization	0.333333	3	0.2	1			
8	Sum	3.500	18.000	1.825	9.333			
9								
10	*NORMALIZED*							
11		Cost	Service	Sophistication	Customization	Average	Consistency Measure	
12	Cost	0.286	0.333	0.274	0.321	0.304	4.0713	
13	Service	0.048	0.056	0.068	0.036	0.052	4.0108	
14	Sophistication	0.571	0.444	0.548	0.536	0.525	4.0869	
15	Customization	0.095	0.167	0.110	0.107	0.120	4.0229	
16						CI =	0.016	
17								
18						RI=	0.9	
19								
20						C. Ratio=	0.018	

Cell	Formula	Copy To
B5	= 1/C4	—
B6	= 1/D4	—
B7	= 1/E4	—
C6	= 1/D5	—
C7	= 1/E5	—
D7	= 1/E6	—
B8	= SUM(B4:B7)	C8:E8
B12	= B4/B$8	B12:E15
F12	= AVERAGE(B12:E12)	F13:F15
G12	= MMULT(B4:E4,$F$12:$F$15)/F12	G13:G15
G16	= AVERAGE(G12:G15)/3	—
G20	= G16/G18	—

FIGURE 12.29

Weighted Average AHP Ratings Using AHP Weights

B3 = =Weights!F12

	A	B	C	D	E
1			*Alternatives' Ratings*		
2	*Criteria*	***Weights***	**RTC**	**PSS**	**EC**
3	**Cost**	0.304	0.315	0.082	0.602
4	**Service**	0.052	0.341	0.593	0.065
5	**Sophistication**	0.525	0.455	0.455	0.091
6	**Customization**	0.120	0.221	0.685	0.093
7					
8	***Wtd Ratings***		0.378	0.376	0.245

Cell	Formula	Copy To
B3	= WEIGHTS!F12	B4:B6
C3	= TOTAL COST!E12	—
D3	= TOTAL COST!E13	—
E3	= TOTAL COST!E14	—
C4	= SERVICE!E12	—
D4	= SERVICE!E13	—
E4	= SERVICE!E14	—
C5	= SOPHISTICATION!E12	—
D5	= SOPHISTICATION!E13	—
E5	= SOPHISTICATION!E14	—
C6	= CUSTOM!E12	—
D6	= CUSTOM!E13	—
E6	= CUSTOM!E14	—
C8	= SUMPRODUCT($B$3:$B$6,C3:C6)	D8:E8

Mark sees that Sophistication of the math algorithms gets the most weight (52.5% in cell F14), followed by Cost (30.4% in cell F12) based on the pairwise comparisons. Again, he's pleased that his consistency measures are close to 4 and therefore that his CI and CR are close to zero.

The final step is to calculate the weighted average ratings of each decision alternative and use the results to decide from which vendor to purchase the new software package. This last step is just like the simple example we gave at the beginning of this section, and Mark pulls the results from all of his other worksheets in order to make this calculation (see the "Comparison" sheet in the same COMPUTER.XLS). This is shown in Figure 12.29 From these results, Mark sees that RTC (.378 in cell C8) barely edges out PSS (.376 in cell D8) for the new software contract, while EC remains a distant third.

12.6 NOTES ON IMPLEMENTATION

As is true of most types of quantitative models, heuristic approaches are typically implemented with the spreadsheet or some other computer program. One difference, in practice, between using heuristic procedures and using more formal models such as linear or quadratic programming, is that in the latter case the computer software already exists. In the heuristic case, however, the application is often *ad hoc*, which implies that the software must be constructed. A typical application of heuristics is, as stated earlier, the area of large combinatorial models, for which obtaining a solution either by enumeration or by applying a formal mathematical or integer programming model would be prohibitively expensive. In all applications of heuristics there is an implicit managerial judgment that "acceptability" rather than "optimality" is an appropriate way of thinking. In other words, it is felt that "good solutions" as opposed to "optimal solutions" can be useful and satisfactory. *This philosophy is particularly well suited to models that are rather vague in their statement, such as high-level models with surrogate objectives or for which there may be numerous conflicting criteria of interest and for which, consequently there is not a clear, definitive single-objective function.*

In practice, the use of heuristics is in some cases closely linked to the field of *artificial intelligence*, where the computer is programmed with heuristic techniques to prove theorems, play chess, and even write poems.

Perhaps the most common use of heuristics in management science has been, to date, in models of assembly-line balancing, job-shop scheduling, and resource allocation in project management. However, recently there has been an increase in the scope of applications to such areas as media selection in marketing, political districting, scheduling of university classes, or positioning urban systems.

In the implementation of all heuristic models, managerial interaction and feedback must play perhaps an even greater role than in the case of more formal modeling, for in the heuristic case the manager must assess not only the model but, implicitly, the heuristic algorithm as well. This assessment is necessary because, *for the same model, different heuristics will lead to different "solutions."*

This close interaction between the model and the decision-maker is also manifest in goal programming when the decision-maker must assign priorities to various goals, such as in the form of ordinal ranking (i.e., *absolute priorities*). Goal programming is an intuitively appealing, and in this sense a "heuristic," approach to models with multiple objectives. In goal programming with absolute priorities, the manager must consider carefully the relative importance or utility of his or her goals. Depending on the spreadsheet model output, the decision-maker may wish to change priorities, or even the number of goals, and rerun the spreadsheet model. In other words, just as with LP, sensitivity analysis becomes an important aspect of implementation. Since goal programming is still more or less in its infancy, the field is developing from a theoretical point of view at a rapid rate, and it seems clear that this development will prompt greater use of the technique, especially as sensitivity analysis becomes better understood.

In practice, computer programs do exist for solving large-scale goal programs in the batch processing mode, but typically these are not part of the standard program libraries. For models of modest size, the interactive mode is ideally suited to the sequential technique described in this chapter.

Key Terms

Heuristic Algorithm. An algorithm that efficiently provides good approximate solutions to a given model, often with estimates as to the goodness of the approximation.

Heuristic. An intuitively appealing rule of thumb for dealing with some aspect of a model.

Heuristic Program. A collection of heuristics and/or heuristic algorithms.

Combinatorial Optimization. An optimization model with a finite number of feasible alternatives.

Setup Time. Time required before an activity can begin.

Greedy Algorithm. An algorithm that says that the maximum improvement should be made at each step of a sequential process.

Next Best Rule. Same as the greedy algorithm.

Precedence Relationships. Means that certain activities must be completed before others may begin.

Personnel Loading Chart. A bar chart showing the total number of people required per week in order to carry out a given schedule of activities.

Slack. In the project scheduling context this refers to the maximum amount of time any given activity can be delayed without delaying completion of the overall project.

Goal Programming. Seeks allowable decisions that come as close as possible to achieving specified goals.

Deviation Variables. Variables used in goal programming to measure the extent to which a specified goal is violated.

Goal Interval Constraint. A constraint for which goals are specified by an interval of indifference, rather than by a specific numerical value.

Absolute Priority. A form of goal programming in which goals must be satisfied in a specific order.

Analytic Hierarchy Process (AHP). A procedure that uses pairwise comparisons to make decisions among competing alternatives when there are multiple criteria that are considered important.

Self-Review Exercises

True-False

1. **T F** Heuristic algorithms are guaranteed to be within a specified percentage of optimality at termination.
2. **T F** The optimal solution to a combinatorial optimization model can, in principle, be found by complete enumeration.
3. **T F** An alternative heuristic in the model of scheduling with limited resources is to move forward that activity that contributes most to the overload (i.e., utilizes the largest number of people).
4. **T F** Goal programming is the only quantitative technique designed for use on models with multiple objectives.
5. **T F** Each step in goal programming with absolute priorities introduces a new goal and eliminates from further consideration all current candidates that do not satisfy this new goal as well as possible.
6. **T F** Consider the goal constraint $12x_1 + 3x_2 + u_1 - v_1 = 100$. Suppose that, because of other constraints in the model, the goal cannot be achieved. If u_1 is positive, the goal is overachieved.
7. **T F** One way to state priorities among goals is to place weights on deviation variables.
8. **T F** Consider the goal interval constraint $180 \le 4x_1 + 12x_2 \le 250$. A correct goal formulation is

$$4x_1 + 12x_2 - v_1 \le 250$$
$$4x_1 + 12x_2 - u_1 \ge 180$$

9. **T F** If a goal interval constraint cannot be achieved (exactly satisfied), then one deviation variable will be positive, and the constraint in which that variable appears will be active.
10. **T F** In goal programming a system constraint is not permitted to be violated.
11. **T F** A goal programming model cannot be infeasible.

Multiple Choice

12. If changeover time of n jobs on a single machine is sequence-dependent, the problem of minimizing total setup time requires the inspection of

a. n sequences
b. 1 sequence
c. $n!$ sequences
d. $\binom{n}{2}$ sequences

13. The intuitively appealing notion that motivates a *greedy* algorithm is to

a. get as close as you can to the optimal solution
b. do the best you can at the current step
c. minimize the number of steps required
d. none of the above

14. In the facility scheduling model, subtracting the minimum setup time in a column from the other entries in that column

a. is a heuristic based on the notion that it is relative costs that matter
b. is guaranteed to yield an optimal solution if the greedy algorithm is applied
c. makes the greedy algorithm not useful
d. all of the above

15. If a goal programming model includes the constraint $g_1(x_1, \ldots, x_n) + u_1 - v_1 = b_1$ and the term $6u_1 + 2v_1$ in the objective function, the decision-maker

a. prefers $g_1(x_1, \ldots, x_n)$ to be larger than, rather than smaller than, b_1
b. prefers $g_1(x_1, \ldots, x_n)$ to be smaller than, rather than larger than, b_1
c. is indifferent as to whether $g_1(x_1, \ldots, x_n)$ is larger than or smaller than b_1

16. Models with multiple objectives

a. are difficult because it is often true that improving one objective will hurt another
b. are difficult because the objectives may be in noncommensurate units (i.e., the problem of "combining apples and oranges")
c. can sometimes be treated with the goal programming approach
d. all of the above

Questions 17, 18, 19 apply to the following problem:

1. $g_1(x_1, x_2) \le b_1$ is a system constraint
2. minimizing underachievement of $g_2(x_1, x_2) = b_2$ is top priority
3. minimizing overachievement of $g_3(x_1, x_2) = b_3$ is next in priority

17. The first step of the solution procedure is
- **a.** Min u_2, s.t. $g_1(x_1, x_2) \le b_1$; $g_2 - u_2 = b_2$; $x_1, x_2, u_2 \ge 0$
- **b.** Min u_2, s.t. $g_1(x_1, x_2) \le b_1$; $g_2 + u_2 \ge b_2$; $x_1, x_2, u_2 \ge 0$
- **c.** Min u_2, s.t. $g_1(x_1, x_2) \le b_1$; $g_2 - u_2 \le b_2$; $x_1, x_2, u_2 \ge 0$

18. Let FR I denote the points (x_1, x_2) obtained in the first step of the solution procedure. The second step is
- **a.** Min $u_3 + v_3$, s.t. (x_1, x_2) in FR I and $g_3(x_1, x_2) + u_3 - v_3 = b_3$
- **b.** Min u_3 , s.t. (x_1, x_2) in FR I and $g_3(x_1, x_2) + u_3 \le b_3$
- **c.** Min v_3, s.t. (x_1, x_2) in FR I and $g_3(x_1, x_2) - v_3 \le b_3$

19. In this model
- **a.** at least one goal will be achieved
- **b.** if the first goal is not achieved, the second goal will not be achieved
- **c.** none of the above

20. Consider a goal program with the constraint

$$g_1(x_1, \ldots, x_n) - v_1 \le b_1, v_1 \ge 0$$

with v_1 in the objective function. Then
- **a.** the goal is to minimize overachievement
- **b.** if $v_1^* > 0$ then the constraint will be active
- **c.** neither of the above
- **d.** both a and b

Answers

1. F, **2.** T, **3.** T, **4.** F, **5.** T, **6.** F, **7.** T, **8.** F, **9.** T, **10.** T, **11.** F, **12.** c, **13.** b, **14.** a, **15.** a, **16.** d, **17.** b, **18.** c, **19.** c, **20.** d

Skill Problems

12-1. For the minimax scheduling model, find an alternative optimal solution to the one given in Figure 12.12.

Problems 12-2, 12-3, and 12-4 refer to the following example of the so-called *facilities layout model:*

Solomon Farson, a management consultant, has been hired to redo the layout of a small bank. There are four key departments to be taken into consideration: (1) Trusts, (2) Estates, (3) Accounting, (4) Savings. These four departments must be assigned to four locations. The distances between locations are given in Table 12.5. Thus the distance from location 2 to location 4 is 1 unit, from 4 to 1 is 1 unit, and so on. A measure of the two-way "daily flows" between the four key departments is shown in Table 12.6.

The problem is to assign the four departments to the four locations (one department per location) in such a way as to minimize the sum of the distance-weighted daily flows. For example, if we make the assignment of departments to locations as follows: $1 \to 1$, $2 \to 2$, $3 \to 3$, $4 \to 4$, then the objective value will be

weighted two-way cost between facilities 1 and 2 = distance × flow = 2(15) =	30
weighted two-way cost between facilities 1 and 3 = distance × flow = 3(20) =	60
weighted two-way cost between facilities 1 and 4 = distance × flow = 1(16) =	16
weighted two-way cost between facilities 2 and 3 = distance × flow = 3(13) =	39
weighted two-way cost between facilities 2 and 4 = distance × flow = 1(9) =	9
weighted two-way cost between facilities 3 and 4 = distance × flow = 2(19) =	38
total cost =	192

Table 12.5

Distances Between Locations

	LOCATION			
Location	**1**	**2**	**3**	**4**
1	0	2	3	1
2	2	0	3	1
3	3	3	0	2
4	1	1	2	0

Table 12.6

Flows Between Departments

	DEPARTMENT			
Dept.	**1**	**2**	**3**	**4**
1	0	15	20	16
2	15	0	13	9
3	20	13	0	19
4	16	9	19	0

12-2. Suppose Solomon assigned department 1 to location 4, department 2 to location 3, department 3 to location 2, and department 4 to location 1.

(a) What would be the distance between departments?

(b) What would be the total cost of Solomon's assignments?

(c) What is the total number of possible assignments of facilities to locations that Solomon would consider if he were to attack the model by complete enumeration?

(d) For the general model of assigning n facilities to n locations, what is the total possible number of assignments?

12-3. Suppose that department 1 is assigned to location 1. Draw a tree, analogous to Figure 12.1, showing the remaining possible assignments of departments 2, 3, and 4 to locations 2, 3, and 4.

12-4. Still referring to Solomon and his layout model,

(a) How many different pairs of departments can be selected from four departments?

(b) Start from the answer to part (a) of Problem 12-2 to improve the assignment by employing the following Best Pairwise Exchange Heuristic, as described here. (Many heuristics have been proposed in the literature for attacking the facilities assignment model. In one study [see Mojena et. al.] involving 12 facilities, it is reported that achieving a true optimum with a branch-and-bound algorithm required 2 hours on a high-speed computer. In 7 seconds the Best Pairwise Exchange Heuristic produced a proposal that was, in terms of associated objective values, within 3% of the optimum.) How much have you improved your objective function?

Step 1: Find the potential improvement in the objective function associated with each pairwise exchange of departments. For example, if departments 1 and 2 are exchanged, the new assignment will be $1 \rightarrow 3$, $2 \rightarrow 4$, $3 \rightarrow 2$, and $4 \rightarrow 1$. That is, the location of departments 1 and 2 are changed, but departments 3 and 4 remain unchanged.

Step 2: Make the pairwise exchange that results in the largest improvement. Then repeat the procedure until no pairwise exchange will improve the value of the objective function.

12-5. Sam Hull is a marketing manager for a pharmaceutical company. He must assign five detail people to five hospitals. The expected sales are shown in Table 12.7.

(a) Use a greedy heuristic to assign each detail person to each hospital so that total expected sales are maximized.

(b) Use the modified heuristic in Section 12.2 (i.e., after transforming the data by subtracting the maximum sales in each column from all other entries in that column, then use the greedy heuristic) to arrive at a new solution. How much better does this heuristic perform than the one used in part (a)?

Table 12.7

	HOSPITAL				
Detail Person	**A**	**B**	**C**	**D**	**E**
1	25	18	23	22	16
2	20	21	18	15	12
3	23	19	20	21	20
4	30	26	25	22	20
5	28	22	23	20	18

12-6. Three jobs—J1, J2, and J3—are to be machined on a lathe. The cost of setting up for a job depends on the setup for the previous job. The cost for changeovers is given in Table 12.8. Currently the lathe is not set up for any job.

(a) Use the greedy heuristic to schedule the jobs. The objective is to minimize the total setup cost.

(b) Use the modified heuristic in Section 12.2 to schedule the jobs.

(c) Does the modified heuristic always produce a better result than the greedy one?

12-7. Erma McZeal is in charge of quality control for the city of Chicago's water supply. There are currently three test stations located in Lake Michigan. Letting (x_1, x_2) denote coordinates in miles, the three existing locations are placed as follows:

station 1: $x_1 = 2, x_2 = 10$

station 2: $x_1 = 6, x_2 = 6$

station 3: $x_1 = 1, x_2 = 3$

Erma's job is to locate a new station in such a way as to minimize the total distance of the new station from the three existing stations. Assume that, because of existing channel marker locations, distance is measured rectangularly. In other words, if the new station is located at $(x_1 = 3, x_2 = 4)$, then it is a distance of $|3 - 2| + |4 - 10|$, or 7 (= 1 + 6) units, from station 1; and so on. Let (x_1, x_2) denote the coordinates of the new station. Formulate a goal programming model to solve Erma's problem.

12-8. Figure 12.30 is the precedence diagram for the activities in a project. The time and the personnel required for each activity are given in Table 12.9. Use the workload smoothing heuristic to generate a schedule for this project.

Table 12.8

	J1	J2	J3
No Setup	\$50	\$35	\$39
J1	—	\$30	\$34
J2	\$41	—	\$30
J3	\$35	\$25	—

FIGURE 12.30

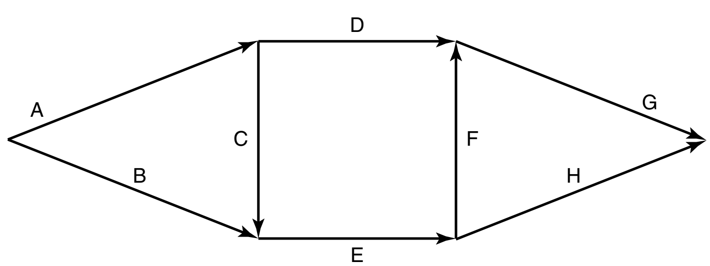


Table 12.9

ACTIVITY	TIME REQUIRED	PERSONNEL
A	1	4
B	2	5
C	1	3
D	2	2
E	2	7
F	1	7
G	1	5
H	1	4

12-9. *Product Mix.* A firm produces two products. Each one must be processed through two machines, each of which has available 240 minutes of capacity per day. Each unit of product 1 requires 20 minutes on machine 1 and 12 minutes on machine 2. Each unit of product 2 requires 12 minutes on machine 1 and 20 minutes on machine 2. In determining the daily product mix, management would like to achieve the following goals:

1. Joint total production of 12 units
2. Production of 9 units of product 2
3. Production of 10 units of product 1

Suppose that management wishes to minimize the underachievement of each of these goals and that predetermined priority weights w_1, w_2, and w_3 are to be assigned to the three goals, respectively. Formulate this as a goal programming model.

12-10. T & C Furniture Company (TCFC) manufactures tables and chairs. Write the goal constraints for the following objectives (let the variables T and C represent the number of tables and chairs, respectively, produced in a period):

(a) A table takes 10 hours to make and a chair 5 hours. The total number of work hours available per period is 3,200. Though idle time and overtime are acceptable, TCFC would like the total number of work hours to be as close to 3,200 as possible.

(b) A table uses one side of wood and a chair half a side; 300 sides of wood are available for a period and no more can be bought. TCFC would like to use as much of this wood as possible in one period.

(c) TCFC makes tables to order and is committed to providing 200 tables in a period. Extra tables, if produced, have to be held in inventory, and the company would like to minimize the number of tables held in inventory.

(d) The demand for chairs is uncertain, but is estimated to be between 200 and 250. The company would like to produce chairs as close to this range as possible.

12-11. Consider the goal programming model:

$$
\begin{aligned}
\text{Min } & P_1 v_1 + P_2 v_2 + P_3 u_3 + P_4(u_4 + v_4) \\
\text{s.t. } & x_2 + u_1 - v_1 = 100 \\
& x_1 + x_2 + u_2 - v_2 = 80 \\
& x_2 + u_3 = 40 \\
& x_1 + 2x_2 + u_4 - v_4 = 160 \\
& x_1, x_2, u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4 \geq 0
\end{aligned}
$$

(a) Use the graphical method to solve the model.

(b) Interpret the third goal $x_2 + u_3 = 40$.

(c) Replace $x_2 + u_3 = 40$ with $x_2 + u_3 \geq 40$. What is the new interpretation?

(d) Use the graphical method to solve the model with the replacement prescribed in (c).

12-12. Consider the goal programming model:

$$
\begin{aligned}
\text{Min } & P_1 v_1 + P_2 v_2 + P_3 v_3 + P_4(u_4 + v_4) \\
\text{s.t. } & x_2 + u_1 - v_1 = 100 \\
& x_1 + x_2 + u_2 - v_2 = 80 \\
& x_1 - v_3 = 40 \\
& x_1 + 2x_2 + u_4 - v_4 = 160 \\
& x_1, x_2, u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4 \geq 0
\end{aligned}
$$

(a) Use the graphical method to solve the model.

(b) Interpret the third goal $x_1 - v_3 = 40$.

(c) Replace $x_1 - v_3 = 40$ with $x_1 - v_3 \leq 40$. What is the new interpretation?

(d) Use the graphical method to solve the model with the replacement prescribed in (c).

12-13. Consider the following goal program:

$$\begin{aligned} \text{Min } & P_1u_2 + P_2v_1 + P_3u_3 \\ \text{s.t.} \quad & x_1 + x_2 + u_1 - v_1 = 80 \\ & x_1 + u_2 - v_2 = 80 \\ & x_2 + u_3 \geq 45 \\ & x_1, x_2, u_1, v_1, u_2, v_2, u_3 \geq 0 \end{aligned}$$

(**a**) Solve by the graphical method.
(**b**) Is the first-priority goal achieved?
(**c**) What about the second and third?
Note: In case of underachievement or overachievement, state actual numerical amounts of the violations.

12-14. Al transportation company operates warehouses and distributes goods to retail outlets. Al has warehouses at five different locations and has four retail customers. The transportation costs per unit, the demands, and the costs of operating the warehouses are given in Table 12.10. All warehouses have unlimited capacity. Al would like to decide which warehouses should be operated and which should be closed. The greedy open heuristic for doing this consists of opening the warehouse that saves the most money and continuing to do this as long as money can be saved.
(**a**) Use the greedy open heuristic to solve this model. Which warehouses are opened?
(**b**) How much money is saved?

12-15. There are six jobs to be processed on two machines (cutting and grinding). Each job must go through the cutting machine before being processed on the grinding machine. Assume that the sequence in which jobs are processed is the same on both machines.

Table 12.11 shows the time (in hours) required to finish a job on each machine. The objective is to schedule the jobs so that the time required to finish all jobs is minimized.
(**a**) How many alternatives should you compare for complete enumeration?
(**b**) What is the time required to finish all jobs if the jobs are processed in the ascending order of the total processing time?
Note: A figure like Figure 12.7 may be helpful. In this application keep all tasks assigned to machine 1 in one row and those assigned to machine 2 in a second row.

12-16. Use AHP to help Mick Mott pick the university that he ought to attend for graduate school. He has two schools that have offered him scholarships (Harvard and Stanford) and has determined that

Table 12.10

	RETAILER				
Warehouse	**A**	**B**	**C**	**D**	**Fixed Cost of Operating Warehouse**
1	5	4	1	6	31
2	9	7	3	5	35
3	8	1	7	4	20
4	4	3	6	2	29
5	6	3	5	2	38
Demand	10	15	6	5	

Table 12.11

	TIME REQUIRED FOR JOB (HOURS)					
Machine	**A**	**B**	**C**	**D**	**E**	**F**
Cutting	3	4	2	1	5	3
Grinding	2	5	2	1	3	4
Total	5	9	4	2	8	7

FIGURE 12.31

	A	B	C
1			
2			
3		Harvard	Stanford
4	Harvard	1	2
5	Stanford	0.5	1

there are four criteria (scholarship amount, prestige, cost to live there, quality of town) that are important to him. See COLLEGE.XLS on your student disk with the following data shown in Figure 12.31.

(a) What are the average ratings for the "Prestige" criterion?
(b) What are the average weights for each criterion?
(c) Which university would you recommend that Mick attend in the fall?

12-17. Wyoming Bentonite Inc. has three deposits of bentonite that are mined for the production of cat litter. Bentonite is a clay-like substance found in central Wyoming that has good absorption properties. The three mines operated by Wyoming Bentonite have slightly differing characteristics of bentonite as determined by calcium and sodium content. Increased calcium results in a whiter color, while increased sodium leads to better absorbency. The customer is concerned with cost, absorbency, and whiteness. The customer specification for whiteness is between 67.2 and 67.8. The specification for absorbency is between 200 and 275. The characteristics of bentonite from each mine are given in Table 12.12.

Management requires a minimum 15% of the blend to come from each mine.

(a) Find the minimum cost for the blend.
(b) If cost is not as important as whiteness and absorbency, find the blend that will place the blend characteristics as close to the center of the customer specifications as possible. What is the cost of that blend?

12-18. CD's.com is an Internet retailer of music. They are trying to maximize revenue prior to their IPO. They have decided to do this through a large marketing campaign in Denver, which is the corporate headquarters. The marketing director believes that the company will be able to gain the most revenue by targeting the teenage customer group first, then the twenty-something, and finally the thirty-something age groups. The three biggest radio stations in Denver have given her the latest listener survey information, shown in Table 12.13.

The company would like to reach 35,000 teens, 28,000 twenty-somethings and 20,000 thirty-somethings.

(a) Find the minimum cost solution that meets the teen listener goal only.
(b) Find the minimum cost solution that meets the teen goal, plus the goal of meeting the twenty-something listener group.
(c) Find the minimum cost solution that meets the goal of all three listening groups.
(d) What is the cost of parts (a), (b), and (c)?

Table 12.12

MINE	WHITENESS	ABSORBENCY	COSTS/TON
A	67.1	175	50
B	68.3	410	110
C	67.7	180	95

Table 12.13

	EXPOSURE			
Station	Teens	20s	30s	Cost/min
KFOX	1,300	1,610	1,042	$157
KDOG	1,537	1,236	1,389	$136
KKAT	535	637	957	$117

Table 12.14

	TO								
		1	2	3	4	5	6	7	8
FROM	1		17	15	9	9	6	3	1
	2	9		13	2	13	4	5	11
	3	17	1		5	6	4	6	8
	4	4	10	16		2	4	3	15
	5	3	14	13	6		2	8	16
	6	15	5	16	9	4		6	5
	7	13	4	4	11	7	14		6
	8	5	5	9	2	6	9	9	

12-19. A beer delivery company has a route with seven deliveries to be made. It originates from the brewery (location 1). Assume that each delivery consumes five minutes. The travel time between stops is given in Table 12.14. Assume that the truck can hold a maximum of four deliveries.

(a) Use the greedy algorithm to determine the route that would minimize driving time to all seven of the delivery locations (this obviously will include a return trip to the brewery to pick up the last three deliveries).

(b) If a larger truck could be obtained so that five deliveries could be completed before returning to the brewery, how would the minimum driving time change?

(c) Could a better solution be obtained with a better algorithm?

12-20. Carol has a project to complete for a new Internet retailer. She has determined the number of software writers needed to implement each activity in the project as well as the precedence of the activities (see Table 12.15).

Use the workload smoothing heuristic to smooth the use of personnel on the project.

12-21. Consider the setup times for the NC machines at Stamped Metal Parts Inc. There are four jobs to be done and each job will require different setup times based on which job had been previously performed (see Table 12.16). The reason for the different setup times is related to the tools and cutting bits that are loaded on the machine for each job. Job zero is the initial unloaded machine condition and is the starting point for the work.

Use the greedy algorithm to minimize total setup time for the four jobs.

Table 12.15

ACTIVITY	PREDECESSORS	TIME (WEEKS)	PEOPLE
I	—	6	6
II	—	4	3
III	—	2	3
IV	III	2	3
V	III	2	3
VI	V	8	5
VII	VI	1	3
VIII	I, II, IV, VII	4	4

Table 12.16

		TO JOB			
		1	2	3	4
	0	50	29	35	42
FROM JOB	1		15	54	36
	2	36		24	42
	3	34	27		37
	4	48	55	58	

12-22. An electronic manufacturing firm assembles security alarms for the home security market. They manufacture three systems—the Guard Dog, the Home Guard, and the top-of-the-line Terminator III. Assembly of the Guard Dog takes 1.5 hours per unit. The Home Guard requires 2 hours to assemble and the Terminator III requires 2.5 hours for assembly. The total available hours of production are 240. The profit contribution of the Guard Dog is $320, the Home Guard is $320, and the Terminator III contributes $350 to the firm's profits. The sales force has predicted next week's sales to be 60 units of each product. Management has specified several goals that are equally important to the firm:

1. Produce 60 of each product.
2. Use all available assembly hours.
3. Generate at least $3,600 in profit.

Formulate and solve the goal-programming problem.

12-23. The marketing manager of a new steak house in Odessa, Texas, has determined that an advertising campaign is needed to boost sales. The owner of the steakhouse and the marketing manager has determined that a budget of $75,000 would be the maximum that could be spent to raise the revenue of the restaurant. *Permian Eats* is a local restaurant magazine that would be ideal for advertising the steak house. *Permian Eats* will sell a full-page ad for $1,000 and the ad exposure is estimated to be 60,000. KOIL, the local TV station, will sell a 30-second advertisement for $6,000 and each ad is estimated to get 600,000 exposures. The manager would like to run at least five TV ads and ten magazine ads. He also would like to spend less than his budget to ensure funds will be available for other alternatives if the ad campaign does not produce the intended results. He has the following goals:

1. Exposures of at least 35,000,000.
2. Spend less than $60,000.
(a) Create a spreadsheet model to meet the goals.
(b) Discuss the results.

12-24. Gert's Sports Emporium is a rapidly growing sporting goods retailer on the East Coast. Bob, the owner of Gert's, has acquired a sizable amount of capital to open new stores in the Chicago area. He can build three types of stores. Superstores (SS) cost $3.5 million to build and employ 150 people. Mall outlet (MO) stores cost $1.7 million and employ 65 people, and an Internet store (IS) would cost Gert's $1 million and would employ 50 people. Gert's has $10 million in capital to invest in the stores but also has multiple goals to meet with the capital investment. Gert's would like to maximize return on investment as well as maximize the number of employees. The expected returns for a superstore, a mall outlet, and an Internet store are $1 million, $500,000, and $1 million, respectively. The number of each type of store is limited by the demographics of the region. The maximum number of Internet stores is one, the maximum number of superstores is three, and mall outlet stores are limited to seven. Evaluate the situation using AHP. Assume there are two criteria, Return and Employees. Bob has rated both criteria using the scale presented in the text. The results are given in Tables 12.17 and 12.18.

(a) Using these ratings, find the best decisions for store mix according to the AHP algorithm.
(b) Evaluate Bob's consistency in rating the alternatives.

Table 12.17

RETURN			
	SS	**MO**	**IS**
SS	1	4	7
MO	0.25	1	5
IS	0.142857	0.2	1

Table 12.18

EMPLOYEES			
	SS	**MO**	**IS**
SS	1	0.25	0.333333
MO	4	1	0.5
IS	3	2	1

Table 12.19

CREDIT TERMS	Big Bank	Little Bank	Bucks R Us
Big Bank	1	2	0.143
Little Bank	0.5	1	6.000
Bucks R Us	7	0.167	1

Table 12.20

CUSTOMER SERVICE	Big Bank	Little Bank	Bucks R Us
Big Bank	1	0.25	1
Little Bank	4	1	0.5
Bucks R Us	1	2	1

Table 12.21

	STICKS SUPPLY	PUCK'S HOUSE	RINKS INC.	GOAL TENDERS
Sticks Supply	1	3	1	0.5
Puck's House	0.333333333	1	0.5	0.25
Rinks Inc.	1	2	1	1
Goal Tenders	2	4	1	1

12-25. Three alternatives are available for the purchase of financial services for Gert's inventory expansion to support new stores in the Chicago area (see Problem 12-24). The suppliers all have different advantages in credit terms and customer service. Bob has rated both criteria (see Tables 12.19 and 12.20).

Use AHP to determine the sole source of financial services for Gert's.

12-26. Bob, the owner of Gert's Sports Emporium (see Problem 12-24), is looking for suppliers of hockey equipment. He is expecting a large jump in sales due to unexpectedly cold winter conditions. He has determined that the decision for a supplier will be based on the ability to provide on-time delivery. He has four suppliers from which to choose and he has developed the ratings shown in Table 12.21.

(a) Use AHP to pick the two best suppliers.

(b) Is Bob consistent with his ratings?

Application Problems

12-27. Given the job-scheduling exercise in Problem 12-15, can you see any improvement when you apply the following heuristic method?

Step 1: List the jobs along with their processing times on the cutting and grinding machines.

Step 2: Find the job with the smallest processing time. If the smallest time is on the cutting machine, schedule the job as early as possible; if it is on the grinding machine, schedule it as late as possible. Break ties arbitrarily.

Step 3: Eliminate the job from the list.

Step 4: Repeat steps 2 and 3 until all jobs have been scheduled.

12-28. The city of Chicago is considering two projects. Each unit of Project A costs $400, generates 20 jobs, and returns $200 at the end of the year. Each unit of Project B costs $600, generates 40 jobs, and returns $200. The city planner would like to achieve the following goals:

1. Keep total expenditure at or below $2400.
2. Generate at least 120 jobs.
3. Maximize return at end of year.

Suppose that the three goals are in order of descending absolute priority.

(a) Use graphical analysis to find the optimal number of units to engage in each project.

(b) Are the goals achieved? If not, what are the underachievements?

(c) What are the net expenditure and the number of jobs generated?

FIGURE 12.32

B6 = =1/D4

	A	B	C	D
1				
2				
3		Bakersfield	Fresno	Oildale
4	Bakersfield	1	4	0.5
5	Fresno	0.25	1	7
6	Oildale	2	0.14285714	1

FIGURE 12.33

	A	B	C	D	E
1					
2					
3		Regal	Camry	Accord	
4	Regal	1	0.33333333	0.2	
5	Camry	3	1	0.5	
6	Accord	5	2	1	
7					

12-29. Another heuristic for solving Problem 12-14 is called the greedy close. In this case we start with all warehouses open and close the one that saves the most money. We continue to do this until we cannot close a warehouse without losing money.

(a) Solve Problem 12-14 using the greedy close heuristic.

(b) Is this solution better or worse than that in Problem 12-14?

12-30. Use AHP to help Marlene Wyatt pick her first job out of college. She has three offers of employment (one in Bakersfield, California; one in Fresno, California; and one in Oildale, California) and has determined that there are three criteria (salary, stability of job, quality of town) that are important to her. See JOB.XLS on your student disk, which contains the following data shown in Figure 12.32.

(a) What are the average ratings for the "Salary" criterion?

(b) Is Marlene consistent? How might you change the comparisons so that she is consistent?

(c) What are the average weights for each of the criterion?

(d) Which job would you recommend that Marlene take?

12-31. Use AHP to help Charles Shumway pick his brand-new automobile. He has narrowed it down to three choices (Buick Regal, Toyota Camry, and Honda Accord) and has determined that there are three criteria (price, Consumer Reports' rating on reliability, speed/performance) that are important to him. See CAR.XLS on your student disk with the following data shown in Figure 12.33.

(a) What are the average ratings for the "Speed" criterion?

(b) What are the average weights for each criterion?

(c) Is Charles consistent with his weights?

(d) Which car would you recommend that Charles buy?

12-32. Suppose that you have been hired by the city council of Peoria, Illinois, to help them meet their overall tax goals. They have three goals (listed in order of descending priority):

1. Limit the tax burden on Lower Income (LI) people to $1.75 billion.

2. Keep the property tax rate under 1%.

3. Minimize the "flight to the suburbs" by keeping the tax burden on Middle Income (MI) people less than $2.5 billion and keeping the tax burden on High Income (HI) people to less than $1.25 billion.

4. Try to eliminate the food and drug sales tax if possible.

The city currently levies five types of taxes: (a) property taxes (where p is the tax rate), (b) sales tax on general items except food and drugs and durable goods (where s is the general sales tax rate), (c) sales tax on food and drugs (f is the sales tax rate on food and drugs), (d) sales tax on durable goods (d is the sales tax rate on durable goods), and (e) gasoline tax (g is the gasoline tax rate).

Relevant information on the revenue generated by a 1% tax is provided in Figure 12.34 (and in the workbook PEORIA.XLS) for each type of tax by category of income people (e.g., LI, MI, or HI). Assume that 10% of the LI people will move out of the city into the suburbs if their tax burden exceeds $1.75 billion, 20% of the MI people will move out of the city into the suburbs if their tax bur-

FIGURE 12.34

	A	B	C	D	E	F	G	H	I	J
1			<---Type of Tax--->							
2	INCOME LEVEL	Property	Sales	Food&Drug	Durable	Gasoline				
3	Low (LI)	400	300	200	50	120				
4	Middle (MI)	1500	450	125	30	80		*Table indicates the millions of*		
5	High (HI)	1200	150	80	15	60		*dollars raised by a 1% tax*		

den exceeds $2.5 billion, and 30% of the HI people will move out of the city into the suburbs if their tax burden exceeds $1.25 billion. You must work within the following "hard" constraints:

- The sales tax rate must be between 1% and 3%, as indicated in the Application Capsule.
- The total revenue raised must exceed the current level of $6.0 billion.
- The tax burden on the HI people can't exceed $1.5 billion.
- The tax burden on the MI people can't exceed $3.0 billion.

(a) Use goal programming to formulate this model.
(b) Which goals can you meet?
(c) What about the ones you can't?

12-33. The Art Institute of Chicago is having a sculptor design and build a large, 80-foot tall, stainless steel sculpture within the building. Because the ventilation is inadequate for engine-driven welding machines, the workers must use large electrically driven welders that are supplied by the building's electrical circuits. Each welder requires 100KW of power and is operated by one person. The sculptor has identified the structural parts of the sculpture that must be completed in the specified order for safety purposes. The sculpture depicts a mother and father watching their son building a tree house. The tree supports a rope ladder, a tire swing, and the tree house floor with the boy standing on the tree house floor. The mother, father, and tree can all be simultaneously welded. After the tree is completed, then both the tire swing and the rope ladder can be added as well as the tree house floor. When the tree house floor has been successfully welded into the tree, then the boy can be welded to the floor. The number of welders required and the time required for each part of this sculpture are given in Table 12.22.

The Art Institute is limited in the amount of electricity that can be supplied to the crew by a contract with the electric utility. No more than 900KW can be supplied to the project at any time. The project must be completed in nine weeks to ensure that the sculpture is ready for the Picasso festival.

Use the workload smoothing heuristic to ensure that the project can be completed without exceeding the electrical capacity of the Art Institute.

12-34. Use the data from Problem 12-21. Is there a better solution for minimizing the setup time at Stamped Metal Parts?

12-35. Use the data from Problem 12-24 (Gert's Sports Emporium).
(a) Use linear programming to determine the store mix that has the maximum return.
(b) Determine the store mix that employs the most people.
(c) Modify the linear program to minimize underachievement of both goals equally.

12-36. A refinery makes diesel, gasoline, and asphalt. The profit contribution for diesel is $3 per barrel. The contribution for gasoline is $2.50 per barrel, and the contribution for asphalt is $3.50 per barrel. The refinery wants to maximize profit, but must also maintain a safe work place. Past experience has shown that the production of asphalt has a "lost work" accident twice as often as the production of diesel or gasoline. The accident rate for the production of asphalt is 0.2 accidents per million barrels produced. The refinery is limited to 10 million barrels of product next month. Five million barrels of gasoline must be produced for an important customer.

Formulate the goal-programming problem to minimize underachievement of both the profit and safety goals equally.

Table 12.22

ACTIVITY	TIME (WEEKS)	PEOPLE
Tree	3	6
Father	2	3
Mother	1	3
Rope Ladder	1	3
Floor	2	6
Boy	4	5
Tire Swing	2	3

Case Study Sleepmore Mattress Manufacturing: Plant Consolidation[1]

W. Carl Lerhos, special assistant to the president of Sleepmore Mattress Manufacturing, had been asked to study the proposed consolidation of plants in three different locations. The company had just added several new facilities as a result of the acquisition of a competitor; some were in markets currently served by existing facilities. The president knew the dollar savings would be fairly easy to calculate for each location, but the qualitative factors and the trade-offs among them were more difficult to judge. This was the area in which he wanted Carl to spend most of his time.

The major objectives in evaluating a consolidation plan for the sites were to maximize manufacturing benefits, maximize sales benefits, and maximize direct financial benefits. These objectives would be composed of exploiting 13 attributes (see Exhibit 1). After spending some time looking at each attribute individually, Carl and the other officers of Sleepmore ranked them in order of most important to least important. They also added the best and worst possible outcomes for each attribute (see Exhibit 2).

Measurements

In each case, the attributes were assigned a number from 0 to 10, with 10 being the best possible outcome mentioned in Exhibit 2. Each location was in a different region, and each of the three locations involved a decision between two alternatives—consolidate the plants there or keep them separate. The plants produced different product lines. Exhibits 3 to 5 give brief descriptions and scores of the three potential consolidation opportunities. Only the "consolidate" alternatives are scored; in other words, each "keep separate" alternative has a default score of 5 for each attribute. Therefore, the attributes are really scored *relative* to the current situation, in which the plants are separate. The scores Carl assigned were based on subjective assessments after talking with the managers and visiting the sites.

Weights

After Carl had scored each attribute on his scale of 0 to 10, he faced the more difficult task of deciding how important one attribute was compared with another. The quantitative attributes would be fairly easy to weigh. He knew that the company's discount rate (15%), along with its planning horizon (10 years), might help in this regard, but he was not quite sure how.

He had heard the president say, "The smaller a plant, the easier it is to manage. If we could improve from a $35 million plant size to a $15 million plant, the gain would be equivalent to a savings from the status quo of $1 million a year in operating costs." Carl made a quick mental calculation that suggested the weight for plant size would be one-half the weight for annual savings—he'd have to check it later though.

The mattress-manufacturing industry required a lot of space. If a consolidation required a new plant or a significant addition, the hassle of moving, as well as hidden expenses, would be additional negative factors. The cost would be $25/sq ft for each additional sq ft of space.

To help him in assigning weights to the other, more qualitative attributes, Carl pulled out his notes from a meeting attended by the president, the vice president of operations, and the vice president of human resources. At this meeting, held at the time of the acquisition, the list shown in Exhibit 2 had been generated and the relative importance of each attribute had been discussed.

The vice president of human resources had said, "Labor is the most important, because the quality of labor determines the major aspects of plant performance (like quality, profitability, etc.). Experience has shown that a good labor force can overcome many obstacles, but a poor labor force leads to trouble. In fact, I think labor is twice as important as the average of all 13 attributes." Carl wondered about the context for this statement. He verified that the vice president had the ranges of Exhibit 2 in mind: Improving labor relations from "create hostile union" to "eliminate hostile union" was twice as valuable as improving the average attribute from worst to best.

The vice president of operations agreed with the comment about labor and said, "I think quality and service, although slightly less important than labor, are two other attributes that deserve more weight than average."

EXHIBIT 1 Hierarchy of Objectives

- I. Maximize Manufacturing Benefits
    - A. Labor
    - B. Management effectiveness
        - 1. Talent availability
        - 2. Plant size
    - C. Operability
        - 1. Product-line complexity
        - 2. Training
        - 3. Production stability
    - D. Facilities
        - 1. Layout
        - 2. Location
        - 3. Space availability
- II. Maximize Sales Benefits
    - A. Maximize service
    - B. Maximize quality
- III. Maximize Direct Financial Benefits
    - A. Minimize initial cost
    - B. Maximize ongoing benefit

[1] This case is to be used as the basis for class discussion rather than to illustrate either the effective or ineffective handling of an administrative situation. Preview Darden case abstracts on the World Wide Web at www.darden.virginia.edu/publishing.

EXHIBIT 2 Thirteen Attributes Selected for Evaluation of Consolidation

RANK	ATTRIBUTE	WORST OUTCOME	BEST OUTCOME
1	Labor	Create hostile union	Eliminate hostile union
2	Quality	Drastically worsen quality	Strongly improve quality
3	Service	Lose business	Increase business
4	Annual savings	Lose $1 million/yr	Save $1 million/yr
5	Initial cost	Cost $5 million	Save $5 million
6	Management talent	Severely worsen management	Strongly improve management
7	Plant size (sales)	Create $35 million plant	Create $15 million plant
8	Plant location	Move from rural area to city	Move from city to rural area
9	Product-line complexity	Increased to full product line	Reduce product line
10	Space availability	Need a new facility (100,000 sq ft)	Save an expansion of 100,000 sq ft
11	Production stability	Increase demand variability	Decrease variability
12	Training	Train all new labor	Small layoff—no new training
13	Plant layout	Create poor layout	Eliminate poor layout

EXHIBIT 3 Consolidation Evaluated at Site 1: Merge Plant 1A into Plant 1B

ATTRIBUTE	PLANT 1A	PLANT 1B	SCORE FOR COMBINING
Labor	Poor	Excellent	9; large improvement
Quality	Poor	Good	9
Service	Poor	Good	8
Annual savings	High overhead	Efficient; merger saves $1 MM/yr	—
Initial cost	Save $1MM if plant merged	N/A	—
Management talent	Poor	Excellent	9
Plant size (sales)	$3 million	$27 million	—
Plant location	Large city	Rural area	10
Product-line complexity	2 major product lines	2 separate lines	0; very complex
Space availability	N/A	Has extra space; needs 0 new sq ft	—
Production stability	Small demand/ high uncertainty	Large demand/low uncertainty	7; reduce variation
Training	N/A	Extra labor available	7.5
Plant layout	Congested plant	Well laid out	7.5

EXHIBIT 4 Consolidation Evaluated at Site 2: Put Plant 2B into Plant 2A

ATTRIBUTE	PLANT 2A	PLANT 2B	SCORE FOR COMBINING
Labor	Average	Poor	6
Quality	Average	Average	5
Service	Average	Good	7
Annual savings	Undercapacity; merger saves $500K	Undercapacity	—
Initial cost	N/A	Save $1MM if merge plant	—
Management talent	Average	Good	6
Plant size (sales)	$5 million	$10 million	—
Plant location	Industrial park	Large city	6
Product-line complexity	2 major product lines	2 different lines	0; very complex
Space availability	Need to add 50K sq ft if merge	No room	—
Production stability	Small demand/high uncertainty	Countercyclical demand	9; reduce variation
Training	Underutilized labor	Underutilized labor	9; small layoff
Plant layout	Excellent	Poor	9

EXHIBIT 5 Consolidation Evaluated at Site 3: Put Plant 3B into Plant 3A

ATTRIBUTE	PLANT A	PLANT B	SCORE FOR COMBINING
Labor	Below average	Good	3; may lose Plant 3A labor
Quality	Average	Average	5
Service	Average	Good	6
Annual savings	Undercapacity; merger saves $200K/yr	Efficient	—
Initial cost	N/A	Save $2MM if merge	—
Management talent	Average	Below average	6
Plant size (sales)	$9 million	$18 million	—
Plant location	Large city	Suburb	4
Product-line complexity	2 major product lines	2 different lines	0; very complex
Space availability	Need 30K sq ft if merge	No room	—
Production stability	Small demand/ high uncertainty	Uncertain demand	6; demand not countercyclical
Training	Underutilized labor	N/A	3; some labor quits
Plant layout	Good	Cramped	7

There seemed to be a consensus that management was the next most important qualitative attribute because, like labor, management would determine the fate of the plant. Unlike labor, however, management could be rather easily changed. Overall, this attribute was considered "average" in terms of importance.

The president then argued for consideration of plant location: "Plant location is as important as plant size. Our data show that plants in more congested areas (cities) tend to be less profitable than plants in rural areas."

The vice president of operations said, "Because Sleepmore produces a different product line in different plants, consolidations could drastically increase complexity and reduce long-term efficiency. I move that product-line complexity be considered the next most important qualitative attribute, albeit its importance is about two-thirds the importance of management talent, in my opinion."

The remaining three attributes—stability, training, and layout—were agreed to have individual effects that were relatively small, but their combined effect was considered about twice that of product-line complexity.

The hardest task was to evaluate the trade-offs that management would be willing to make between quantitative and qualitative factors. In this regard, the president had expressed difficulty to Carl in choosing between a situation with initial cost savings of $7 million and a situation where a hostile union was eliminated.

Decision

Carl had to figure out an effective way to combine all this information about both quantitative and qualitative factors to make decisions whether to consolidate at *each* of the three sites. He wondered how sensitive his decisions would be to the weights he assigned each attribute.

References

Colin Benjamin, Ike Ehie, and Yildirim Omurtag, "Planning Facilities at the University of Missouri–Rolla," *Interfaces*, 22, no. 4 (1992): 95–105.

Abraham Charnes and William Cooper, *Management Models and Industrial Applications of Linear Programming* (New York: John Wiley, 1961).

William Gavett, "Three Heuristic Rules for Sequencing Jobs to a Single Production Facility," *Management Science*, 11 (1965): B166–76.

Richard Mojena, Thomas Vollmann, and Yoshihiro Okamoto, "On Predicting Computational Time of a Branch-and-Bound Algorithm for the Assignment of Facilities," *Decision Sciences*, 7, no. 4 (1976): 856–867.

Thomas Saaty, *The Analytic Hierarchy Process* (New York: McGraw-Hill, 1988).

Andres Weintraub and Bruce Bare, "New Issues in Forest Land Management from an Operations Research Perspective," *Interfaces*, 26, no. 5 (1996): 9–25.