№15

$a=2,\ b=0,\ c=-4.$

$$p_\xi(x)=\begin{cases}A(|x^3|+1), & -2\le x\le 2\\ 0, & x<-2,\ x>2\end{cases}$$

Решение:

$$|x^3|=\begin{cases}x^3, & x\ge 0\\ -x^3, & x<0\end{cases}$$

$$\Rightarrow \quad p_\xi(x)=\begin{cases}0, & x<-2\\ A(1-x^3), & -2\le x<0\\ A(x^3+1), & 0\le x\le 2\\ 0, & x>2\end{cases}$$

а) $\int\limits_{-\infty}^{\infty} p_\xi(x)\,dx=1$

$$\int\limits_{-2}^{0} A(1-x^3)\,dx+\int\limits_{0}^{2} A(x^3+1)\,dx=1$$

$$A\left(x-\frac{x^4}{4}\right)\Big|_{-2}^{0}+A\left(\frac{x^4}{4}+x\right)\Big|_{0}^{2}=$$

$$=A\left(0-\left(-2-\frac{16}{4}\right)\right)+A\left(\frac{16}{4}+2\right)=$$

$$=A(2+4)+A(2+4)=12A=1\Rightarrow A=\frac{1}{12}$$

$$A=\frac{1}{12}$$

Т. О. $p_\xi(x) = \begin{cases} 0, & x < -2 \\ \frac{1}{12}(1-x^3), & -2 \le x < 0 \\ \frac{1}{12}(x^3+1), & 0 \le x \le 2 \\ 0, & x > 2 \end{cases}$

б) 1) $x < -2 \Rightarrow F_\xi(x) = 0$

2) $-2 \le x < 0 \Rightarrow$

$\Rightarrow F_\xi(x) = \int_{-2}^{x} \frac{1}{12}(1-t^3)\,dt =$

$= \frac{1}{12}\left(t - \frac{t^4}{4}\right)\Big|_{-2}^{x} =$

$= \frac{1}{48}\left(-x^4 + 4x + 24\right)$

3) $0 \le x \le 2 \Rightarrow$

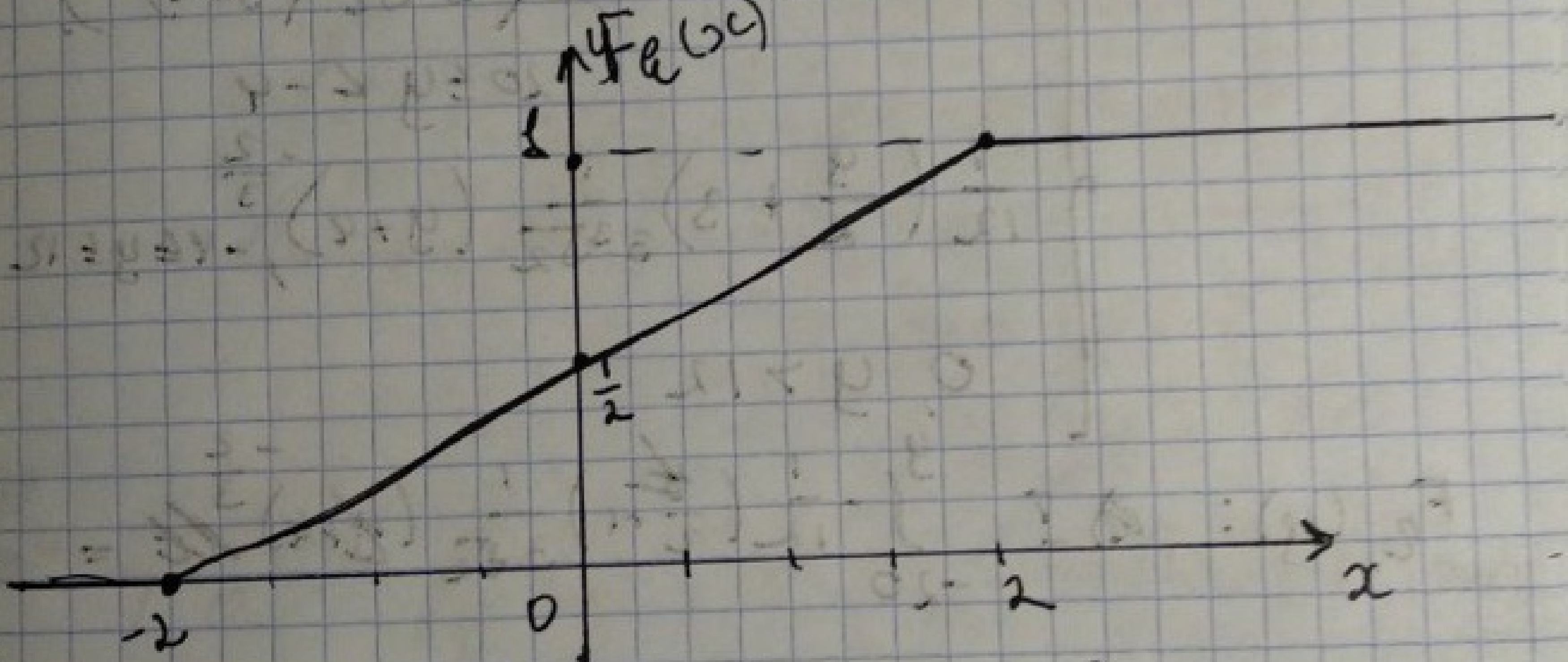
$\Rightarrow F_\xi(x) = \int_{-2}^{0} \frac{1}{12}(1-t^3)\,dt +$

$+ \int_{0}^{x} \frac{1}{12}(t^3+1)\,dt = \frac{1}{2} + \frac{1}{48}(x^4+4x) =$

$= \frac{1}{48}(x^4 + 4x + 24).$

4) $x > 2 \Rightarrow F_\xi(x) = 1$

T.O. $$F_\xi(x) = \begin{cases} 0, & x<-2 \\ \frac{1}{48}\left(-x^4+4x+24\right), & -2\le x<0 \\ \frac{1}{48}\left(x^4+4x+24\right), & 0\le x\le 2 \\ 1, & x>2 \end{cases}$$



в) $\eta = 2(\xi-0)^3-4 = 2\xi^3-4$

$$y = 2x^3-4$$

$$x = \sqrt[3]{\frac{y+4}{2}}$$

$$x' = \frac{1}{3}\cdot\frac{1}{\sqrt[3]{2}}\cdot(y+4)^{-\frac{2}{3}}$$

$x\in[-2,2] \Rightarrow y\in[-20,12],\ y(0)=-4$

1) $x\in[-2,0) \Rightarrow y\in[-20,-4)$ и

$$p_\eta(y) = \frac{1}{12}\left(1-\left(\frac{y+4}{2}\right)\right)\cdot\left|\frac{1}{3\sqrt[3]{2}}(y+4)^{-\frac{2}{3}}\right|$$

2) $x \in [0,2] \Rightarrow y \in [-4, 12]$

$$p_\eta(y) = \frac{1}{12}\left(\frac{y+4}{2}+1\right)\left|\frac{1}{3\sqrt[3]{2}}(y+4)^{-\frac{2}{3}}\right|$$

$$p_\eta(y) = \begin{cases} 0, & y < -20 \\ -\frac{1}{12}\left(\frac{y}{2}+1\right)\left(\frac{1}{3\sqrt[3]{2}}(y+4)^{-\frac{2}{3}}\right), & -20 \le y < -4 \\ \frac{1}{12}\left(\frac{y}{2}+3\right)\frac{1}{3\sqrt[3]{2}}(y+4)^{-\frac{2}{3}}, & -4 \le y \le 12 \\ 0, & y > 12 \end{cases}$$

$$F_\eta(y): \text{I} \int_{-20}^{y} -\frac{1}{12}\left(\frac{t}{2}+1\right)\frac{1}{3\sqrt[3]{2}}(t+4)^{-\frac{2}{3}}dt =$$

$$= -\frac{1}{2\cdot 36\cdot\sqrt[3]{2}}\int_{-20}^{y}(t+2)(t+4)^{-\frac{2}{3}}dt =$$

$$= -\frac{1}{72\sqrt[3]{2}}\cdot\frac{3}{4}(t-4)(t+4)^{\frac{1}{3}}\Big|_{-20}^{y} =$$

$$= \boxed{-\frac{1}{72\sqrt[3]{2}}\frac{3}{4}(y-4)(y+4)^{\frac{1}{3}}+\frac{1}{2}}$$

при $y = -4$; $= 0 + \frac{1}{2}$

3) $h \int_{-20}^{-4} -\frac{1}{12 \cdot 3 \cdot \sqrt[3]{2}} \left(\frac{y}{2}+1\right)(y+4)^{-\frac{2}{3}} dy +$

$x \in [0,2]$
$y \in [-4,12]$ $+ \int_{-4}^{y} \frac{1}{36\sqrt[3]{2}} \left(\frac{t}{2}+3\right)(t+4)^{-\frac{2}{3}} dt =$

$= \frac{1}{2} + \frac{1}{\cancel{72}_{24}\sqrt[3]{2}} \cdot \frac{3}{4}(t+12)(t+4)^{\frac{1}{3}} \Big|_{-4}^{y} =$

$= \boxed{\frac{1}{2} + \frac{1}{96\sqrt[3]{2}}(y+12)(y+4)^{\frac{1}{3}}}$

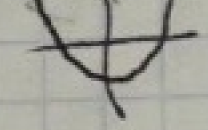
1 при $y=12$

т. о.
$$F_{\eta}(y) = \begin{cases} 0, & y < -20 \\ \frac{1}{2} - \frac{1}{96\sqrt[3]{2}}(y-4)(y+4)^{\frac{1}{3}}, & -20 \le y < -4 \\ \frac{1}{2} + \frac{1}{96\sqrt[3]{2}}(y+12)(y+4)^{\frac{1}{3}}, & -4 \le y \le 12 \\ 1, & y > 12 \end{cases}$$

г) $\mu = 2\xi^2 - 4$

$y = 2x^2 - 4 = 2(x^2 - 2)$

не монотонна на R.



I. ф-я уб. на $(-\infty, 0)$

$$x = -\sqrt{\frac{y+4}{2}}$$

$$x' = -\frac{1}{2\sqrt{2}}(y+4)^{-\frac{1}{2}}$$

$x \in (-\infty; 0) \Rightarrow y \in (-4; +\infty)$

$x \in [-2, 0) \Rightarrow y \in (-4, 4]$

$$p_\mu(y) = \frac{1}{12}\left(1 + \left(-\frac{y+4}{2}\right)^{\frac{3}{2}}\right)\left|-\frac{1}{2\sqrt{2}}(y+4)^{-\frac{1}{2}}\right|$$

$-4 < y \le 4$

II. ф-я возр. на $[0, +\infty)$

$$x = \sqrt{\frac{y+4}{2}}$$

$$x' = \frac{1}{2\sqrt{2}\sqrt{y+4}}$$

$x \in (0; +\infty) = y \in (-4, +\infty)$

$x \in (0; 2] \Rightarrow y \in (-4, 4]$

$$p_\mu(y) = \frac{1}{12}\left(\left(\frac{y+4}{2}\right)^{\frac{3}{2}} + 1\right)\left|\frac{1}{2\sqrt{2}\sqrt{y+4}}\right| \quad -4 < y \le 4$$

~~сложим~~ удвоим на [-4, 4]

Т. о.

$$\cancel{P_\mu} \; P_\mu(y) = \begin{cases} 0, & y \le -4 \\ \dfrac{\left(1 + \left(\frac{y+4}{2}\right)^{\frac{3}{2}}\right)}{12\sqrt{2}\sqrt{y+4}}, & -4 \le y \le 4 \\ 0, & y > 4 \end{cases}$$

$$F_\mu(y) = \int\limits_{-4}^{y} \frac{1 + \left(\frac{t+4}{2}\right)^{\frac{3}{2}}}{12\sqrt{2}\sqrt{t+4}} = \quad , -4 < y \le 4$$

$$= \frac{1}{96}\left(y^2 + 8y + 8\sqrt{2}\sqrt{y+4} + 16\right)$$

Т. о.

$$F_\mu(y) = \begin{cases} 0, & y \le -4 \\ \frac{1}{96}\left(y^2 + 8y + 8\sqrt{2}\sqrt{y+4} + 16\right), & -4 < y \le 4 \\ 1, & y > 4 \end{cases}$$

N16

$\xi \sim N(-5, 4)$, $a_1 = -10$, $a_2 = 2$, $a = 2$, $b = 6$

Решение: $x_1 = 20$, $x_2 = 40$

а) $P\{-10 < \xi < 2\}$ – ?

$\exists$ $\mu = \frac{\xi + 5}{4}$, тогда $\mu \sim Norm(0,1)$

и $F_\mu(x) = \frac{1}{2} + \varphi_0(x)$

$P\{-10 < \xi < 2\} = P\{-10 < 4\mu - 5 < 2\} =$

$= P\{\frac{-10+5}{4} < \mu < \frac{2+5}{4}\} = P\{\frac{-5}{4} < \mu < \frac{7}{4}\} =$

$= \varphi_0(\frac{7}{4}) - \varphi_0(-\frac{5}{4}) \approx \varphi_0(1{,}75) + \varphi_0(1{,}25) =$

$\approx 0{,}45994 + 0{,}39435 = 0{,}85429$

б) $P\{20 < \eta < 40\} = P\{20 < e^{2\xi + 6} < 40\} =$

$= P\{\underbrace{\ln 20}_{\approx 3} < 2\xi + 6 < \underbrace{\ln 40}_{\approx 3{,}69}\} = P\{\frac{3-6}{2} < \xi < \frac{3{,}69-6}{4}\}$

$= P\{-1{,}5 < \xi < -1{,}155\} = P\{\frac{-1{,}5+5}{4} < \mu < \frac{-1{,}155+5}{4}\} =$

$= P\{0{,}875 < \mu < 0{,}971\} =$

$= \varphi_0(0{,}97) - \varphi_0(0{,}88) \approx 0{,}33398 - 0{,}31057 = 0{,}02341$

№17

$n_1 = 6$ б. Изв. $m = 6$ ш.

$n_2 = 4$ ч. ξ - число вын. б. ш. с.в.

$n_3 = 5$ с. η - синих. с.в.

Решение:

а) и б) η от 0 до 5

ξ от 0 до 6.

Всего способов: $C_{15}^6 = 5005$

~~а) $\overset{с.}{C_5^0}\overset{ч.}{C_4^0}C_6^6$~~

~~б) C_6^0~~

$C_6^0 = 1$

$C_6^2 = 15 = C_6^4$

$C_6^3 = 20$

$C_5^2 = 10 = C_5^3$

$C_4^2 = 6$

ξ/η	0	1	2	3	4	5	P_ξ
0	0	0	$\frac{2}{1001}$	$\frac{8}{1001}$	$\frac{6}{1001}$	$\frac{4}{5005}$	$\frac{84}{5005}$
1	0	$\frac{6}{1001}$	$\frac{48}{1001}$	$\frac{72}{1001}$	$\frac{24}{1001}$	$\frac{6}{5005}$	$\frac{756}{5005}$
2	$\frac{3}{1001}$	$\frac{60}{1001}$	$\frac{180}{1001}$	$\frac{120}{1001}$	$\frac{15}{1001}$	0	$\frac{1890}{5005}$
3	$\frac{16}{1001}$	$\frac{120}{1001}$	$\frac{160}{1001}$	$\frac{40}{1001}$	0	0	$\frac{1680}{5005}$
4	$\frac{18}{1001}$	$\frac{60}{1001}$	$\frac{30}{1001}$	0	0	0	$\frac{540}{5005}$
5	$\frac{24}{5005}$	$\frac{6}{1001}$	0	0	0	0	$\frac{54}{5005}$
6	$\frac{1}{5005}$	0	0	0	0	0	$\frac{1}{5005}$
P_η	$\frac{42}{1001}$	$\frac{252}{1001}$	$\frac{420}{1001}$	$\frac{240}{1001}$	$\frac{45}{1001}$	$\frac{2}{1001}$	

$\Sigma = 1$

$\Sigma = 1$

$\Sigma p_{ij} = 1$ (проверено!)

$P\{\xi/\eta=4\}$

$P\{\xi/\eta=5\}$

в)

ξ	0	1	2	3	4	5	6
$P\{\xi/\eta=0\}$	0	0	$\frac{3}{42}$	$\frac{16}{42}$	$\frac{18}{42}$	$\cancel{\frac{24}{88}}$ $\frac{4}{35}$	$\frac{1}{210}$
$P\{\xi/\eta=1\}$	0	$\frac{6}{252}$	$\frac{60}{252}$	$\frac{120}{252}$	$\frac{60}{252}$	$\frac{6}{252}$	0
$P\{\xi/\eta=2\}$	$\frac{2}{420}$	$\frac{48}{420}$	$\frac{180}{420}$	$\frac{160}{420}$	$\frac{30}{420}$	0	0
$P\{\xi/\eta=3\}$	$\frac{8}{240}$	$\frac{72}{240}$	$\frac{120}{240}$	$\frac{40}{240}$	0	0	0
$P\{\xi/\eta=4\}$	$\frac{6}{45}$	$\frac{24}{45}$	$\frac{15}{45}$	0	0	0	0
$P\{\xi/\eta=5\}$	0,4	0,6	0	0	0	0	0

$$\frac{\cancel{24}^{4}}{\cancel{5005}_{5}}\cdot\frac{100}{\cancel{42}_{7}} = \frac{4}{35}$$

$$\frac{\cancel{24}^{4}}{\cancel{5005}_{5}}\cdot\frac{100}{\cancel{42}} \quad 5 \quad 2\cdot 7$$

η	0	1	2	3	4	5
$P\{\eta/\xi=0\}$	0	0	$\frac{10}{84}$	$\frac{40}{84}$	$\frac{30}{84}$	$\frac{4}{84}$
$P\{\eta/\xi=1\}$	0	$\frac{30}{756}$	$\frac{240}{756}$	$\frac{360}{756}$	$\frac{120}{756}$	$\frac{6}{756}$
$P\{\eta/\xi=2\}$	$\frac{15}{1890}$	$\frac{300}{1890}$	$\frac{900}{1890}$	$\frac{600}{1890}$	$\frac{75}{1890}$	0
$P\{\eta/\xi=3\}$	$\frac{80}{1680}$	$\frac{600}{1680}$	$\frac{800}{1680}$	$\frac{200}{1680}$	0	0
$P\{\eta/\xi=4\}$	$\frac{90}{540}$	$\frac{300}{540}$	$\frac{150}{540}$	0	0	0
$P\{\eta/\xi=5\}$	$\frac{24}{54}$	$\frac{30}{54}$	0	0	0	0
$P\{\eta/\xi=6\}$	1	0	0	0	0	0

т.к. $P\{\xi=0, \eta=0\} \neq P\{\xi=0\} \cdot P\{\eta=0\}$

$$\left(0 \neq \frac{84}{5005} \cdot \frac{42}{1001}\right)$$

$\Rightarrow$ ξ и η - зависимые с.в

$$\frac{462+1320}{3003} = \frac{1782}{3003}$$

2) 1) $F_{\xi\eta}(8, 2) = P\{\xi < 8, \eta < 2\} =$

$= P\{\eta < 2\} = P\{\eta = 0\} + P\{\eta = 1\} =$

$= \frac{42}{1001} + \frac{252}{1001} = \frac{294}{1001}$

2) $F_{\xi\eta}(5, 6) = P\{\xi < 5, \eta < 6\} =$

$= P\{\xi < 5\} = 1 - P\{\xi = 5\} - P\{\xi = 6\} =$

$= 1 - \frac{54}{5005} - \frac{1}{5005} = \frac{4950}{5005}$

3) $F_{\xi\eta}(2, 3) = P\{\xi < 2, \eta < 3\} =$

$= P\{\xi = 0, \eta = 2\} + P\{\xi = 1, \eta = 1\} +$

$+ P\{\xi = 1, \eta = 2\} = \frac{2}{1001} + \frac{6}{1001} + \frac{48}{1001} = \frac{56}{1001}$

g)

$$\mu = |\xi - \eta| \cos \pi\eta$$

$$\mu_1 = \xi - 3\eta$$

$$\mu_2 = 2\eta + \xi$$

ξ	η	μ	μ_1	μ_2	
0	2	2	-6	4	
0	3	-2	-9	6	
0	4	2	-12	8	
0	5	-2	-15	10	
1	1	-1	-2	3	-2
1	2	1	-5	5	-5
1	3	-1	-8	7	-8
1	4	1	-11	9	-11
1	5	-1	-14	11	-14
2	0	0	2	2	
2	1	+0	-1	4	
2	2	0	-4	6	
2	3	+0	-7	8	
2	4	0	-10	10	
3	0	1	3	3	
3	1	-1	0	5	
3	2	1	-3	7	
3	3	-1	-6	9	
4	0	2	4	4	
4	1	-2	1	6	
4	2	2	-2	8	
5	0	3	5	5	
5	1	-3	2	7	
6	0	4	6	6	

μ	-3	-2	-1	0	1	2	3	4
P_μ	$\frac{30}{5005}$	$\frac{344}{5005}$	$\frac{1196}{5005}$	$\frac{1890}{5005}$	$\frac{1240}{5005}$	$\frac{280}{5005}$	$\frac{24}{5005}$	$\frac{1}{5005}$

$\Sigma = 1$

$-2: \frac{300 + 4 + 40}{5005} = \frac{344}{5005}$

$200 + 600 + 6 + 360 + 30 = 1196$

$-1:$

$1: 800 + 80 + 120 + 240 = 1240$

$2: 150 + 90 + 30 + 10 = 280$

μ_2

μ_1/μ_2	2	3	4	5	6	7	8	9	10	11
-15	0	0	0	0	0	0	0	0	$\frac{4}{5005}$	0
-14	0	0	0	0	0	0	0	0	~~0~~	$\frac{6}{5005}$
-12	0	0	0	0	0	0	$\frac{8}{1001}$	0	0	0
-11	0	0	0	0	0	0	0	$\frac{24}{1001}$	0	0
-10	0	0	0	0	0	0	0	0	$\frac{15}{1001}$	0
-9	0	0	0	0	$\frac{8}{1001}$	0	0	0	0	0
-8	0	0	0	0	0	$\frac{72}{1001}$	0	0	0	0
-7	0	0	0	0	0	0	$\frac{120}{1001}$	0	0	0
-6	0	0	$\frac{2}{1001}$	0	0	0	0	$\frac{40}{1001}$	0	0
-5	0	0	0	$\frac{48}{1001}$	0	0	0	0	0	0
-4	0	0	0	0	$\frac{180}{1001}$	0	0	0	0	0
-3	0	0	0	0	0	$\frac{160}{1001}$	0	0	0	0
-2	0	$\frac{6}{1001}$	0	0	0	0	$\frac{30}{1001}$	0	0	0
-1	0	0	$\frac{60}{1001}$	0	0	0	0	0	0	0
0	0	0	0	$\frac{120}{1001}$	0	0	0	0	0	0
1	0	0	0	0	$\frac{60}{1001}$	0	0	0	0	0
2	3:1001	0	0	0	0	$\frac{6}{1001}$	0	0	0	0
3	[illegible]	16:1001	0	0	0	0	0	0	0	0
4	0	0	18:1001	0	0	0	0	0	0	0
5	0	0	0	24:5005	0	0	0	0	0	0

1:5005

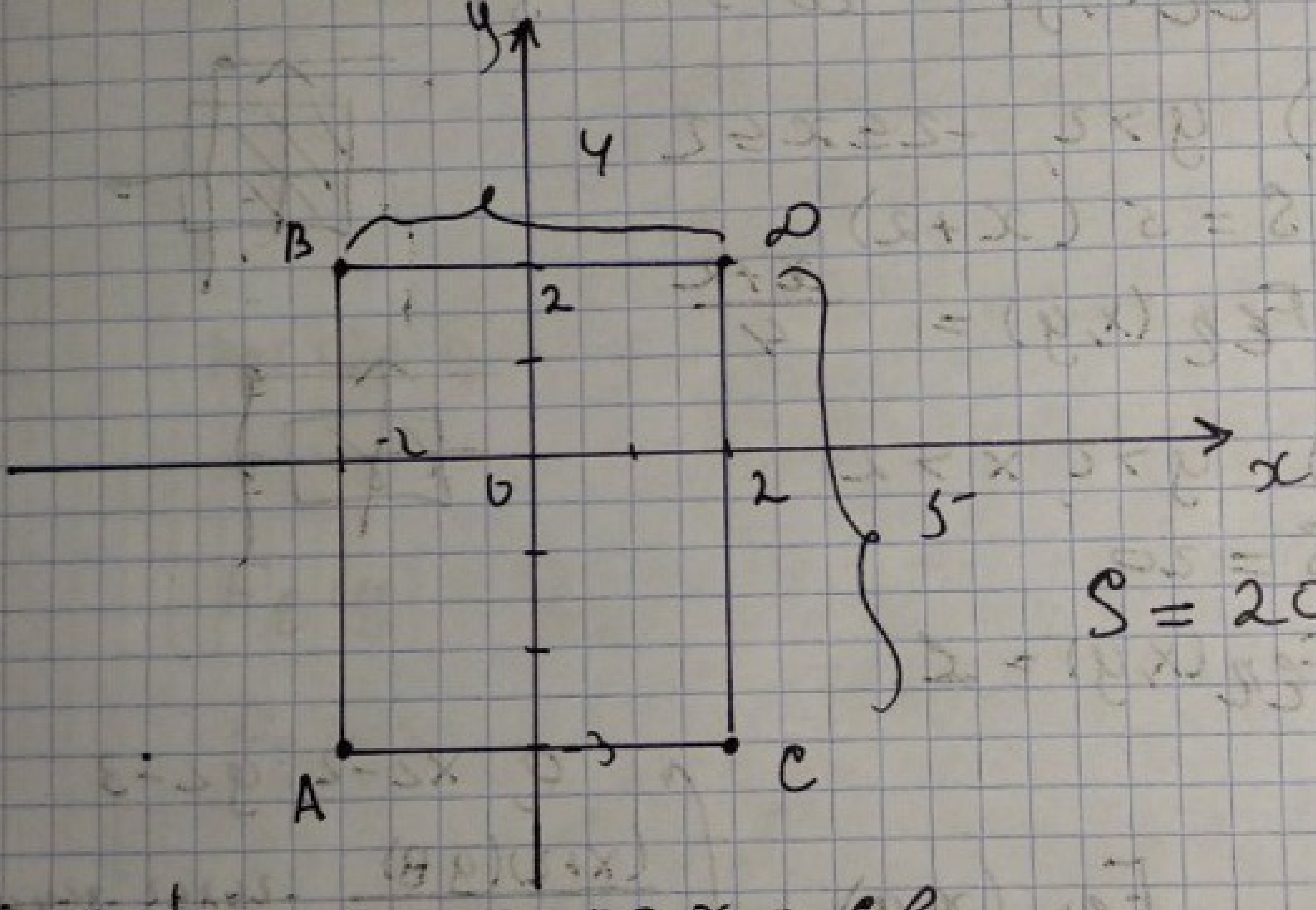
μ_1

$\Sigma=1$

№18

$A(-2,-3)$; $B(-2,2)$; $C(2;-3)$, $D(2;2)$

Решение:



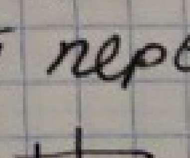
$S = 20$

точки $BADCD$	ξ - коорд. по x, с.в.
	η - коорд. по y, с.в.

$P\{\xi < x, \eta < y\}$ - ?

а) $x < -2$ или $y < -3$ => $F_{\xi\eta}(x,y) = 0$,

т.к. нет перес.



б) $-2 \le x \le 2$, $-3 \le y \le 2$

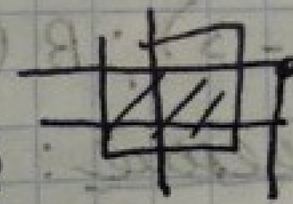
$S = (x+2)(y+3)$

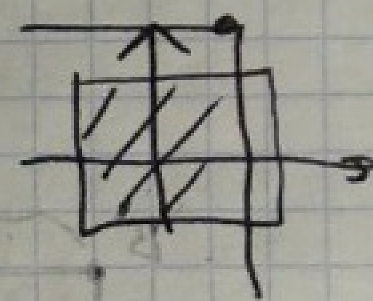
$F_{\xi\eta}(x,y) = \dfrac{(x+2)(y+3)}{20}$

3) $x > 2,\ -3 \le y \le 2$

$S = 4\cdot(y+3)$

$F_{\xi\eta}(x,y) = \frac{4}{20}(y+3) = \frac{y+3}{5}$

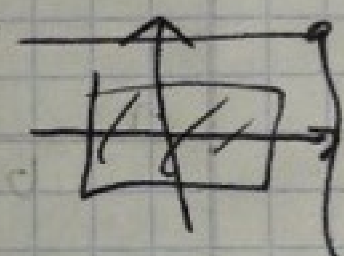




4) $y > 2,\ -2 \le x \le 2$

$S = 5\,(x+2)$

$F_{\xi\eta}(x,y) = \frac{x+2}{4}$



5) $y > 2,\ x > 2$

$S = 20$

$F_{\xi\eta}(x,y) = 1.$

$$F_{\xi\eta}(x,y) = \begin{cases} 0, & x < -2,\ y < -3 \\ \frac{(x+2)(y+3)}{20}, & -2 \le x \le 2,\ -3 \le y \le 2 \\ \frac{y+3}{5}, & x > 2,\ -3 \le y \le 2 \\ \frac{x+2}{4}, & y > 2,\ -2 \le x \le 2 \\ 1, & x > 2,\ y > 2 \end{cases}$$

$$P_{\xi\eta}(x,y) = \begin{cases} 0, & x < -2,\ y < -3 \\ \frac{1}{20}, & -2 \le x \le 2,\ -3 \le y \le 2 \\ 0, & x > 2,\ y > 3 \end{cases}$$

б) $F_\xi(x) = F_{\xi\eta}(x, +\infty)$

$$F_\xi(x) = \begin{cases} 0, & x < -2 \\ \frac{x+2}{4}, & -2 \le x \le 2 \\ 1, & x > 2 \end{cases}$$

$$p_\xi(x) = F'_\xi(x) = \begin{cases} 0, & x < -2,\ x > 2 \\ \frac{1}{4}, & -2 \le x \le 2 \end{cases}$$

$F_\eta(y) = F_{\xi\eta}(+\infty, y)$

$$F_\eta(y) = \begin{cases} 0, & y < -3 \\ \frac{y+3}{5}, & -3 \le y \le 2 \\ 1, & y > 2 \end{cases}$$

$$p_\eta(y) = F'_\eta(y) = \begin{cases} 0, & y < -3,\ y > 2 \\ \frac{1}{5}, & -3 \le y \le 2 \end{cases}$$

в) $p_\xi(x|y) = \dfrac{p_{\xi\eta}(x,y)}{p_\eta(y)} = \begin{cases} \frac{1}{20} \cdot \frac{5}{1} = \frac{1}{4}, & -2 \le x \le 2 \\ 0, & x < -2, x > 2 \end{cases}$

$F_\xi(x|y) = \int\limits_{-\infty}^{x} p_\xi(t|y)\,dt =$

$\begin{cases} 0, & x < -2 \\ = \int\limits_{-2}^{x} \frac{1}{4}\,dt = \frac{x+2}{4}, & -2 \le x \le 2 \\ 1, & x > 2 \end{cases}$

Это всё при $p_\eta(y) \ne 0$, т.е. при $-3 \le y \le 2$

$p_\eta(y|x) = \dfrac{p_{\xi\eta}(x,y)}{p_\xi(x)} = \begin{cases} \frac{1}{20} \cdot \frac{4}{1} = \frac{1}{5}, & -3 \le y \le 2 \\ 0, & y < -3, y > 2 \end{cases}$

$\cancel{F_\xi}\ F_\eta(x/y) = \int\limits_{-\infty}^{y} p_\eta(t|x) -$

Δ_0

$= \begin{cases} 0, & y < -3 \\ \int\limits_{-3}^{y} \frac{1}{5}\,dt = \frac{y+3}{5}, & -3 \le y \le 2 \\ 1, & y > 2 \end{cases}$

т.к. $p_\eta(y) = p_\eta(y|x)$ с.в. ξ и η независимы

Это всё при $-2 \le x \le 2$, т.е. при $p_\xi(x) \ne 0$

$y = -3$

$-5 = -3x$

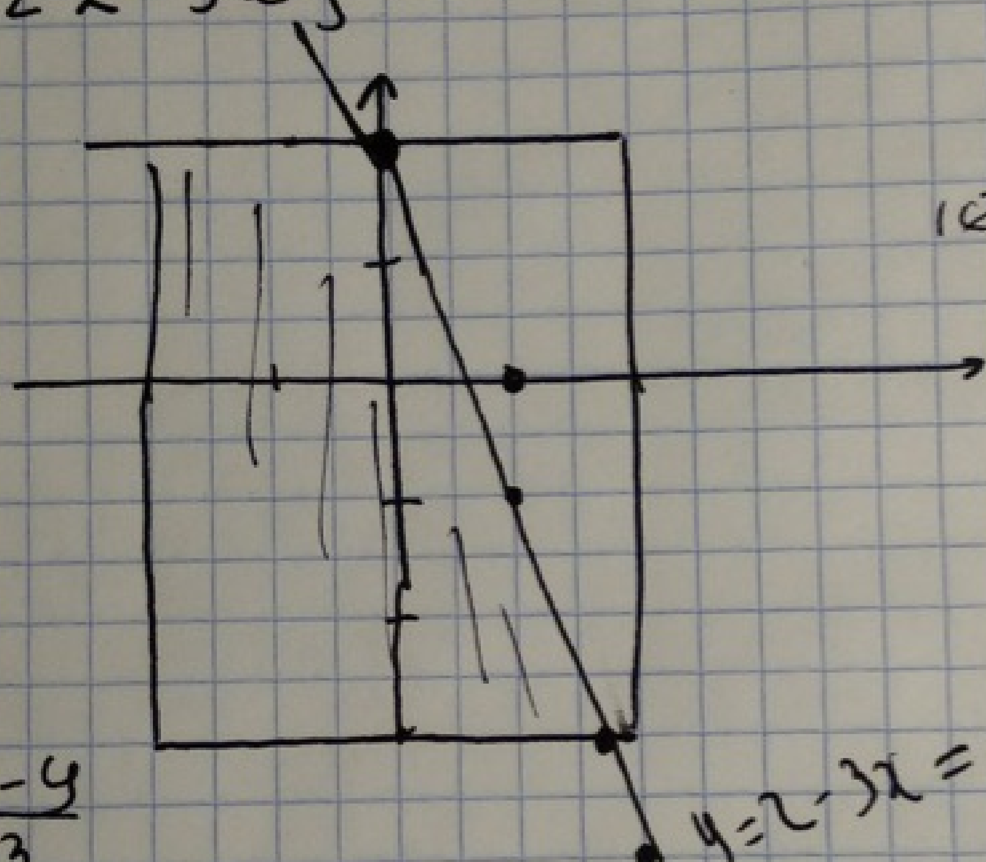
$x = \frac{5}{3}$

2) $\mu = 3\xi + \eta$, $z = 2$

$F_\mu(2) = P\{\mu < 2\} = P\{3\xi + \eta < 2\} =$

$= \iint \frac{1}{20}\, dx\, dy \ (=)$

$D \cap \{y < 2 - 3x\}$



$10 \cdot \frac{1}{2} \cdot 5.$

$y - 2 = -3x$

$y = 2 - 3x \Rightarrow x = \frac{2-y}{3}.$

$-2 \le x \le \frac{2-y}{3}$

$-3 \le y \le 2$

$$(=)\ \frac{1}{20}\int_{-3}^{2} dy \int_{-2}^{\frac{2-y}{3}} dx = \frac{1}{20}\int_{-3}^{2}\left(\frac{2-y}{3} + 2\right) dy =$$

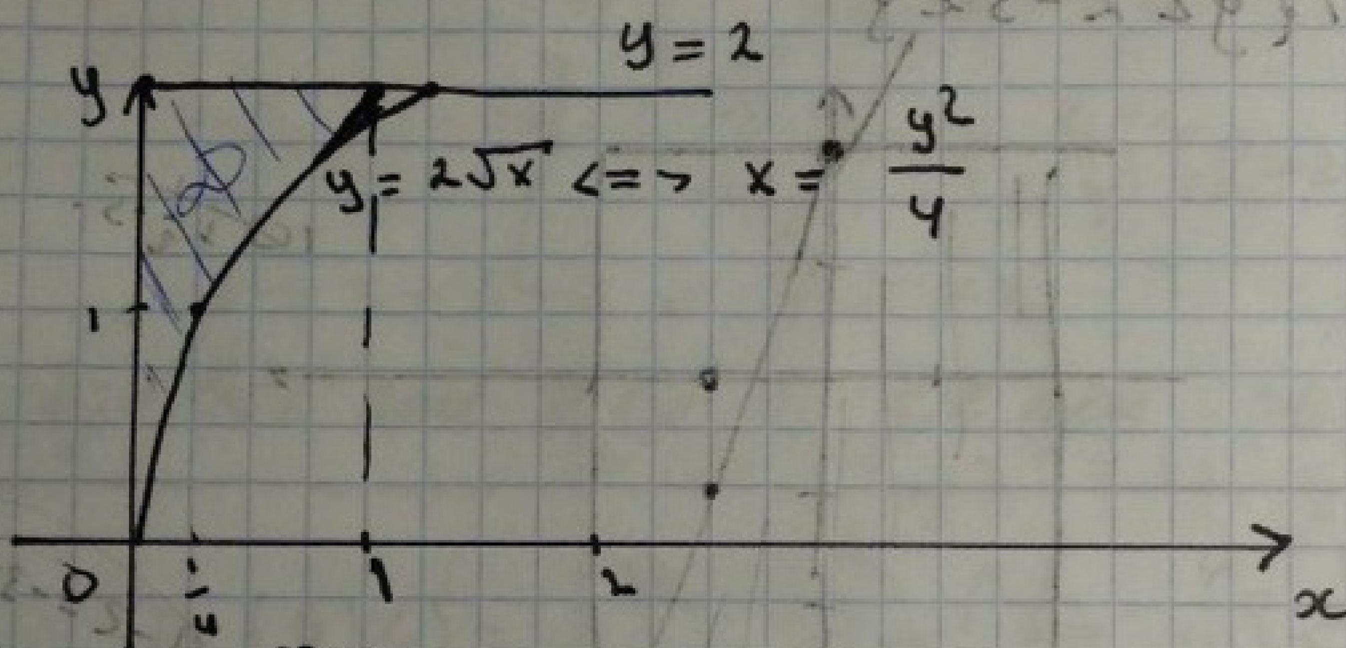
$$= \frac{1}{20}\int_{-3}^{2} \frac{8-y}{3}\, dy = \frac{1}{60}\left(8y - \frac{y^2}{2}\right)\Big|_{-3}^{2} =$$

$$= \frac{1}{60}\left(\overset{14}{16 - 2} - \overset{38}{\left(-24 - \frac{9}{2}\right)}\right) = \frac{1}{60}\left(\frac{85}{2}\right) = \cancel{\frac{97}{120}}\ \frac{85}{120}$$

№19

$$p_{\xi\eta}(x,y) = \begin{cases} C(2x^2+2y), & (x,y)\in D \\ 0, & (x,y)\notin D \end{cases}$$

$D: \{x=0,\ y=2,\ y=2\sqrt{x}$



a) $\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} p_{\xi\eta}(x,y)\,dx\,dy = 1$

$$\iint\limits_D C(2x^2+2y)\,dx\,dy = C\int\limits_0^1 dx \int\limits_{2\sqrt{x}}^{2}(2x^2+2y)\,dy =$$

$$= C\int\limits_0^1 (2x^2y+y^2)\Big|_{2\sqrt{x}}^{2}dx =$$

$$= C\int\limits_0^1 (4x^2+4-4x^2\sqrt{x}-4x)\,dx =$$

$$= C\left(\frac{4x^3}{3}+4x-\frac{2\cdot 4x^{\frac{7}{2}}}{7}-2x^2\right)\Big|_0^1 =$$

$$= C\left(\frac{4}{3}+4-\frac{8}{7}-2\right) = C\left(\frac{28-24}{21}+2\right) = C\left(\frac{4+42}{21}\right) = \frac{46}{21}C$$

$\frac{46}{21}C = 1$

$C = \frac{21}{46}$

$C = \frac{21}{46}$

$\frac{21}{46}(2x^2+2y) = \frac{2\cdot 21}{46}(x^2+y) = \frac{21}{23}(x^2+y)$

Т.о. $p_{\xi\eta}(x,y) = \begin{cases} \frac{21}{23}(x^2+y), & (x,y)\in D \\ 0, & (x,y)\notin D \end{cases}$

б) $F_{\xi\eta}(2,3) = P\{\xi < +2, \eta < 3\} =$

$= \iint\limits_{D\cap\left\{\substack{x<2\\ y<3}\right\}} \frac{21}{23}(x^2+y)\,dx\,dy =$ 

$= 1$, т.к. $D\cap\left\{\substack{x<2\\ y<3}\right\} = D$

$\boxed{F_{\xi\eta}(2,3) = 1.}$

в) 1) $p_\xi(x) = \int\limits_{-\infty}^{\infty} p_{\xi\eta}(x,y)\,dy =$

$= \begin{cases} \int\limits_{2\sqrt{x}}^{2} \frac{21}{23}(x^2+y)\,dy, & 0\le x\le 1 \\ 0, & x<0;\ x>1 \end{cases}$

$p_\xi(x) = \begin{cases} -\frac{42}{23}\left(x^{\frac{5}{2}} - x^2 + x - 1\right), & 0\le x\le 1 \\ 0, & x<0;\ x>1 \end{cases}$

2) $F_\xi(x) = \int\limits_{-\infty}^{x} p_\xi(t)\,dt =$

$$= \begin{cases} 0, & x<0 \\ \int\limits_0^x -\frac{42}{23}\left(t^{\frac{5}{2}} - t^2 + t - 1\right) dt, & 0 \le x \le 1 \\ 1, & x> \end{cases}$$

$$F_\xi(x) = \begin{cases} 0, & x<0 \\ \frac{1}{23}x\left(-12x^{\frac{5}{2}} + 14x^2 - 21x + 42\right), & 0 \le x \le 1 \\ 1, & x>1 \end{cases}$$

3) $p_\eta(y) = \int\limits_{-\infty}^{\infty} p_{\xi\eta}(x,y)\,dx =$

$$= \begin{cases} \int\limits_0^{y^2/4} \frac{21}{23}(x^2+y)\,dx, & 0 \le y \le 2 \\ 0, & y<0;\ y>2 \end{cases}$$

$$p_\eta(y) = \begin{cases} \frac{7y^3(y^3+48)}{1472}, & 0 \le y \le 2 \\ 0, & y<0;\ y>2 \end{cases}$$

4) $F_\eta(y) = \int_{-\infty}^{y} p_\eta(t)\,dt =$

$$= \begin{cases} 0, & y<0 \\ \int_0^y \frac{7t^3(t^3+48)}{1472}\,dt, & 0\le y\le 2 \\ 1, & y>2 \end{cases}$$

$$F_\eta(y) = \begin{cases} 0, & y<0 \\ \frac{y^4(y^3+84)}{1472}, & 0\le y\le 2 \\ 1, & y>2 \end{cases}$$

2) 1) $p_\xi(x|y) = \frac{p_{\xi\eta}(x,y)}{p_\eta(y)} = \frac{\frac{\cancel{81}^{\,3}}{\cancel{23}}(x^2+y)}{\frac{\cancel{7}}{\cancel{1472}_{\,64}}(y^6+48y^3)} =$

$$= \begin{cases} \frac{192(x^2+y)}{y^6+48y^3}, & x\in\left[0;\frac{y^2}{4}\right] \\ 0, & x<0;\ x>\frac{y^2}{4} \end{cases}$$

$\boxed{\text{при } 0\le y\le 2}$

2) $F_\xi(x|y) = \int\limits_{-\infty}^{x} p_\xi(t|y)\,dt =$

$$= \begin{cases} 0, & x<0 \\ \int\limits_0^x \dfrac{192(\cancel{x}^2+y)\,dt}{y^6+48y^3}, & 0 \le x \le \dfrac{y^2}{4} \\ 1, & x > \dfrac{y^2}{4} \end{cases}$$

$$F_\xi(x|y) = \begin{cases} 0, & x<0 \\ \dfrac{64(x^3+3xy)}{y^6+48y^3}, & 0 \le x \le \dfrac{y^2}{4} \\ 1, & x > \dfrac{y^2}{4} \end{cases}$$

при $x = \dfrac{y^2}{4}$

в-е $= 1$.

$\boxed{\text{при } 0 \le y \le 2}$

3) $p_\eta(y|x) = \dfrac{p_{\xi\eta}(x,y)}{p_\xi(x)} = \dfrac{\frac{21}{23}(x^2+y)}{-\frac{42}{23}(x^{\frac{5}{2}}-x^2+x-1)} =$

$$= \begin{cases} -\dfrac{1}{2}\dfrac{(x^2+y)}{(x^{\frac{5}{2}}-x^2+x-1)}, & y \in [2\sqrt{x}, 2] \\ 0, & y < 2\sqrt{x};\ y>2 \end{cases}$$

$\boxed{\text{при } 0 \le x \le 1}$

4) $F_\eta(y|x) = \int\limits_{-\infty}^{y} p_\eta(t|x)\,dt =$

$$= \begin{cases} 0, & y < 2\sqrt{x} \\ \int\limits_{2\sqrt{x}}^{y} -\frac{1}{2}\frac{(x^2+t)\,dt}{(x^{\frac{5}{2}}-x^2+x-1)}, & 2\sqrt{x} \le y \le 2 \\ 1, & y > 2 \end{cases}$$

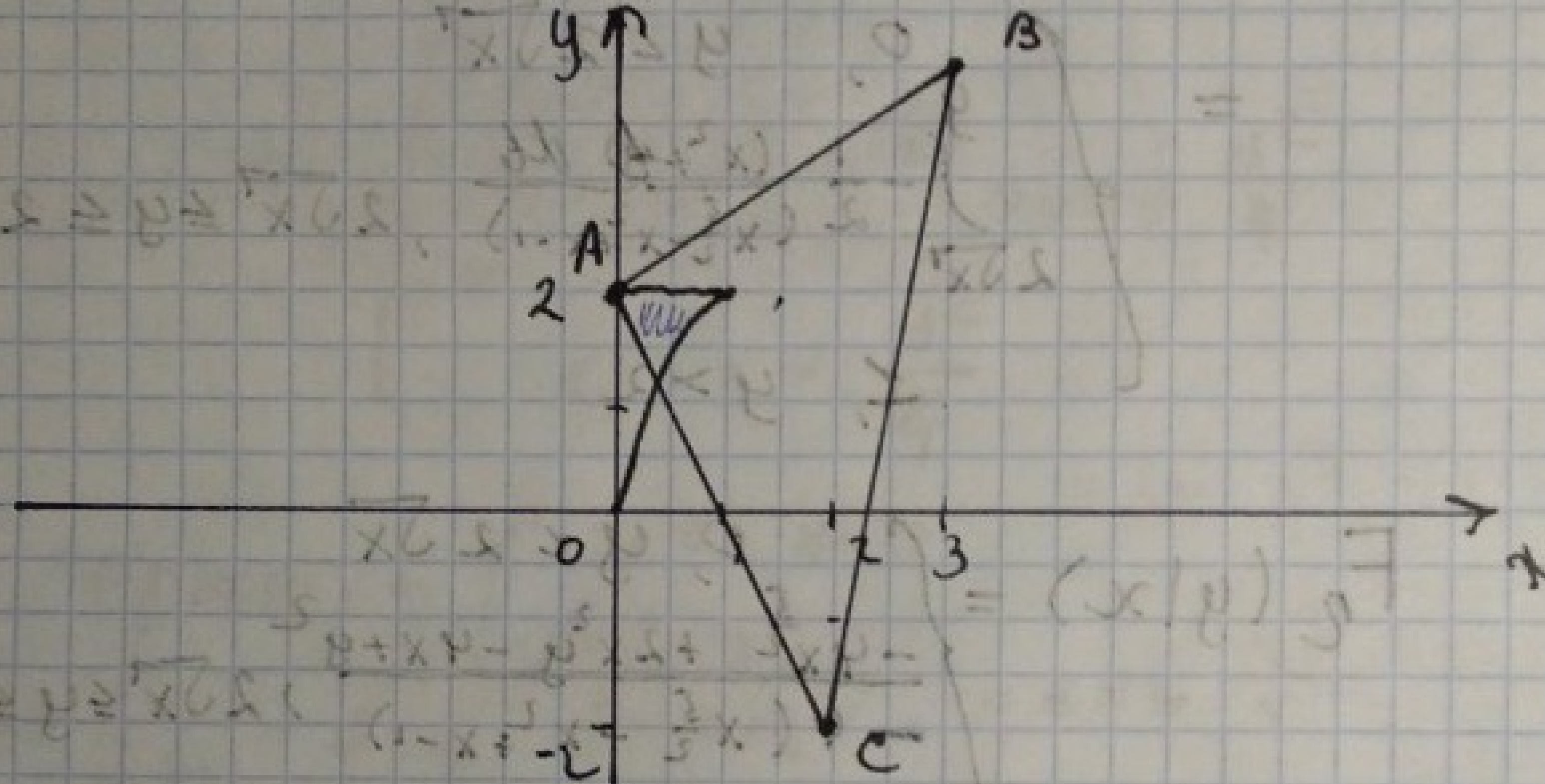
$$F_\eta(y|x) = \begin{cases} 0, & y < 2\sqrt{x} \\ \frac{-4x^{\frac{5}{2}}+2x^2y-4x+y^2}{-4(x^{\frac{5}{2}}-x^2+x-1)}, & 2\sqrt{x} \le y \le 2 \\ 1, & y > 2 \end{cases}$$

$\boxed{\text{при } 0 \le x \le 1}$

5) ~~Т.к. p_ξ~~(

$p_\eta(y|x) \neq p_\eta(y) \Rightarrow$ ξ и η - зависим.

g) A(0,2), B(3,4), C(2,-2)



ур-е AC: $y = kx + b$

$$\begin{cases} 2 = k \cdot 0 + b \\ \cancel{-2 = 2k + b} \end{cases} \quad b = 2 \qquad \cancel{4k = 2,\ k = \frac{2}{4}}$$

$\cancel{y = -\frac{1}{2}x + 2}$ $\quad 2k = -4,\ k = -2$

$$\boxed{y = -2x + 2} \iff x = \frac{2 - y}{2}$$

$$P\{\text{т. в } \Delta\text{ке}\} = \iint\limits_{D \cap \Delta} \frac{21}{23}(x^2 + y)\,dx\,dy \;=$$

$$\frac{2-y}{2} \le x \le \frac{y^2}{4}$$

$$\frac{2-y}{2} = \frac{y^2}{4} \Rightarrow \sqrt{5} - 1 = y$$

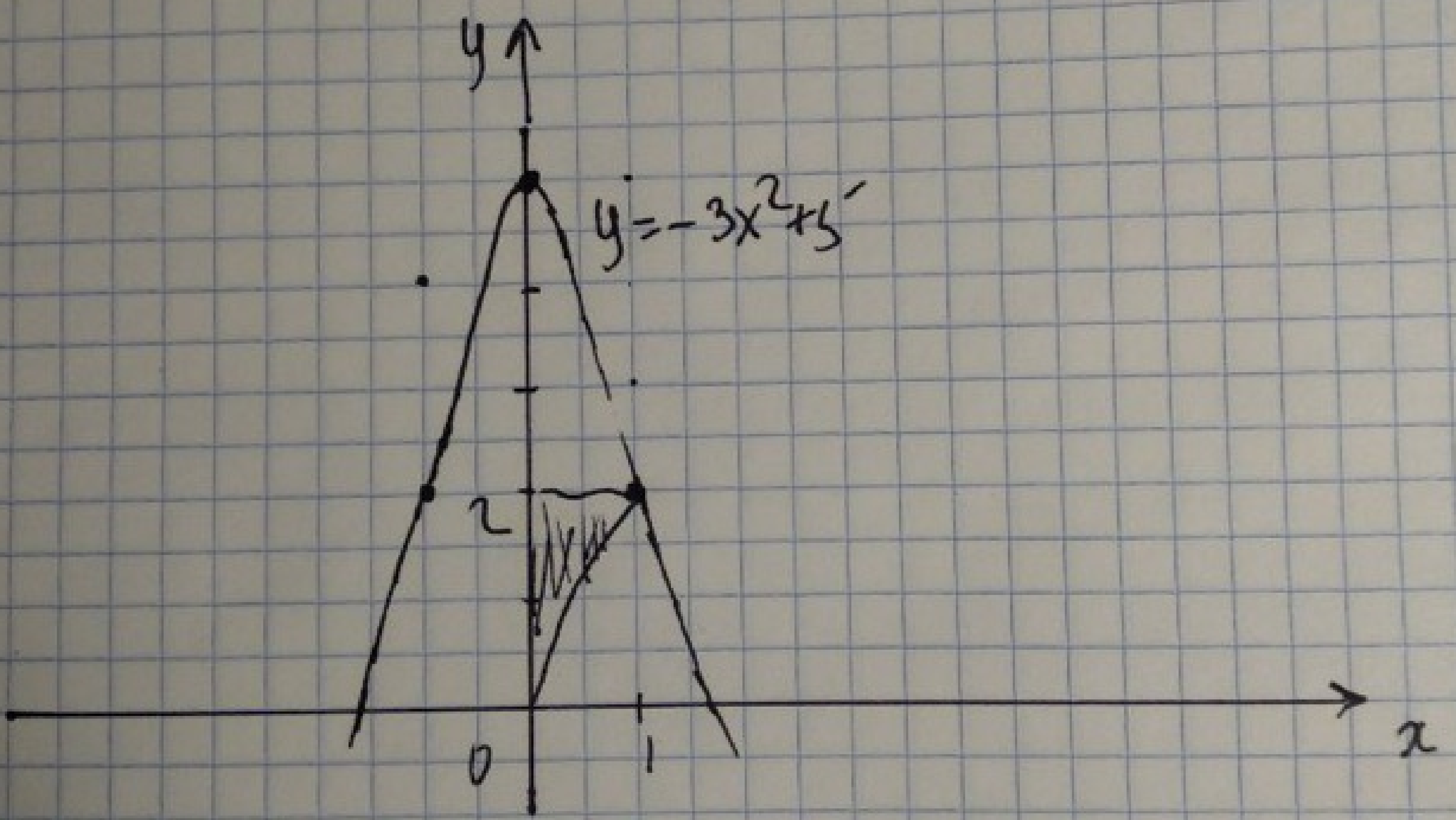
$$\sqrt{5} - 1 \le y \le 2$$

$$P = \frac{21}{23}\int_{\sqrt{5}-1}^{2} dy \int_{\frac{2-y}{2}}^{\frac{y^2}{4}} (x^2 + y)\,dx$$

e) $F_\mu(5) = P\{\mu < 5\} = P\{3\xi^2 + \eta < 5\} =$

$= \iint \frac{21}{23}(x^2 + y)\,dx\,dy$ =

$D \cap \{y < -3x^2 + 5\}$



= $\iint\limits_{D} \frac{21}{23}(x^2 + y)\,dx\,dy = 1$

№20

$$p_\xi(x)=\begin{cases}\cos x, & 0\le x\le \frac{\pi}{2}\\ 0, & x<0,\ x>\frac{\pi}{2}\end{cases}$$

$$p_\eta(y)=\begin{cases}\frac{1}{3}, & 1\le y\le 4\\ 0, & y<1,\ y>4\end{cases}$$

Решение

$$p_\mu(z)=\int_{-\infty}^{\infty} p_\xi(x)\cdot p_\eta(z-x)\,dx = \int_{+0}^{\frac{\pi}{2}}\frac{1}{3}\cos x\,dx$$

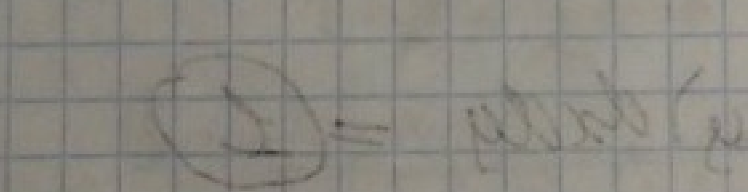
$x\in[0;\frac{\pi}{2}]$ и $z-x\in[1,4]$

$1\le z-x\le 4$

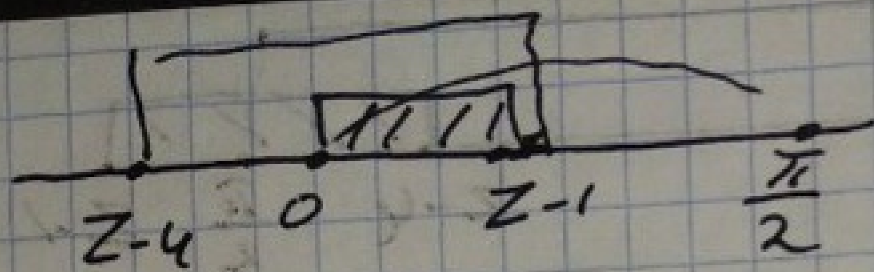
$1-z\le -x\le 4-z$

$z-4\le x\le z-1$

Рассм. все пересеч. $[0;\frac{\pi}{2}]$ и $[z-4;z-1]$

1) 

если $z-1<0 \Leftrightarrow z<1$, то нет пересеч. и $p_\mu(z)=0$

2) 

Если $z-1 \ge 0$ и $z-4<0$,

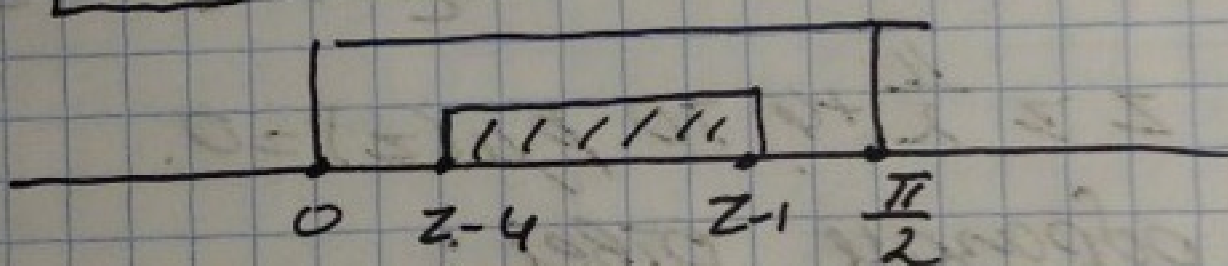
$\Updownarrow$

$1 \le z < 4$, то

$$p_\mu(z) = \int_0^{z-1} \frac{1}{3}\cos x\,dx =$$

$$= \frac{1}{3}\sin x \Big|_0^{z-1} = \frac{1}{3}\big(\sin(z-1) - \underbrace{\sin 0}_{0}\big) =$$

$$= \boxed{\frac{1}{3}\sin(z-1), \quad 1 \le z < 4}$$

3) 

Если $z-4 \ge 0$ и $z-1 < \frac{\pi}{2}$

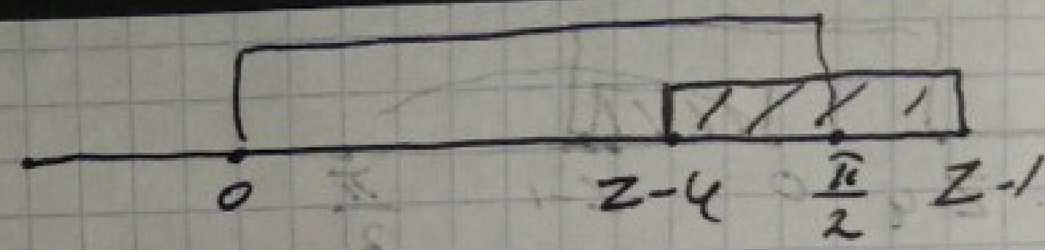
$\Updownarrow$

$4 \le z < \frac{\pi}{2}+1$, то

$$p_\mu(z) = \int_{z-4}^{z-1} \frac{1}{3}\cos x\,dx = \frac{1}{3}\sin x \Big|_{z-4}^{z-1} =$$

$$= \boxed{\frac{1}{3}\big(\sin(z-1) - \sin(z-4)\big)}$$

4)



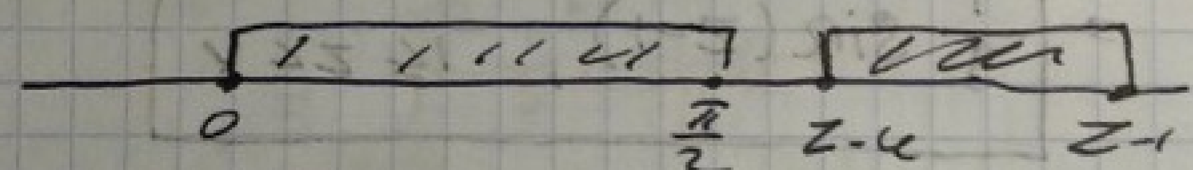
если $\frac{\pi}{2}+1 \le z < \frac{\pi}{2}+4$, то

$$p_\mu(z) = \frac{1}{3}\int_{z-4}^{\frac{\pi}{2}} \cos x\,dx =$$

$$= \frac{1}{3}\sin x \Big|_{z-4}^{\pi/2} = \frac{1}{3}\left(\sin\frac{\pi}{2} - \sin(z-4)\right) =$$

$$= \boxed{\frac{1}{3} - \frac{1}{3}\sin(z-4)}$$

5)



если $z \geq \frac{\pi}{2}+4$, то $p_\mu(z) = 0$.

таким образом, <u>ответ</u>:

$$p_\mu(z) = \begin{cases} 0, & z < 1 \\ \frac{1}{3}\sin(z-1), & 1 \le z \le 4 \\ \frac{1}{3}\left(\sin(z-1) - \sin(z-4)\right), & 4 \le z < \frac{\pi}{2}+1 \\ \frac{1}{3}\left(1 - \sin(z-4)\right), & \frac{\pi}{2}+1 \le z \le \frac{\pi}{2}+4 \\ 0, & z \geq \frac{\pi}{2}+4 \end{cases}$$